# Light Source II

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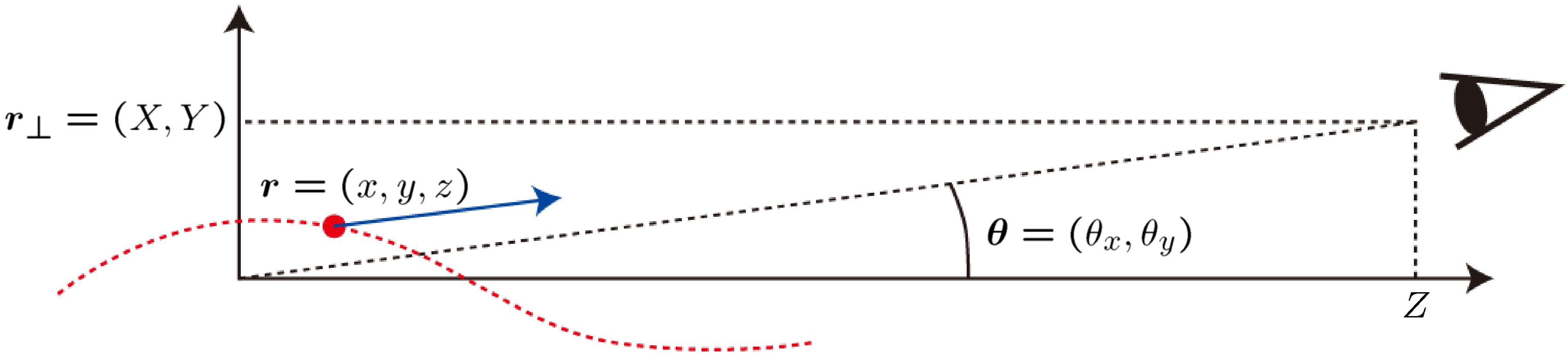
## Characteristics of SR (2)

- Electron Trajectory in the ID
- Qualitative Description of Wiggler Radiation
- Qualitative Description of Undulator Radiation

# Coordinate Systems




SR emitted by an electron moving at ***r*** = *(x,y,z)*
Observation of SR at ***R*** = *(X,Y,Z)*

If the far-field approximation (|***r***|<<*Z*) is applicable, the radiation pattern depends only on the observation angle $\boldsymbol{\theta} = (\theta_x, \theta_y)$.

# Field Integrals

$$\frac{d\boldsymbol{P}}{dt} = m\gamma\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{v}\times\boldsymbol{B} \Rightarrow \begin{cases} m\gamma\dot{v}_x = -e(v_y B_z - v_z B_y) \\ m\gamma\dot{v}_y = -e(v_z B_x + v_x B_z) \end{cases}$$

Equation of motion of an electron moving in a magnetic field ***B***

$$B_z \equiv 0$$

$$m\gamma\frac{dv_{x,y}}{v_z dt} = m\gamma\frac{dv_{x,y}}{dz} = \pm e B_{y,x}$$

$$\beta_{x,y} = \pm\frac{e}{\gamma mc}\int^z B_{y,x}(z')dz' \equiv \pm\frac{e}{\gamma mc}I_{1y,1x}(z)$$

$$x, y = \pm\frac{e}{\gamma mc}\int^z\int^{z'} B_{y,x}(z'')dz'' \equiv \pm\frac{e}{\gamma mc}I_{2y,2x}(z)$$

$I_1, I_2$ : 1st and 2nd field integrals of the ID

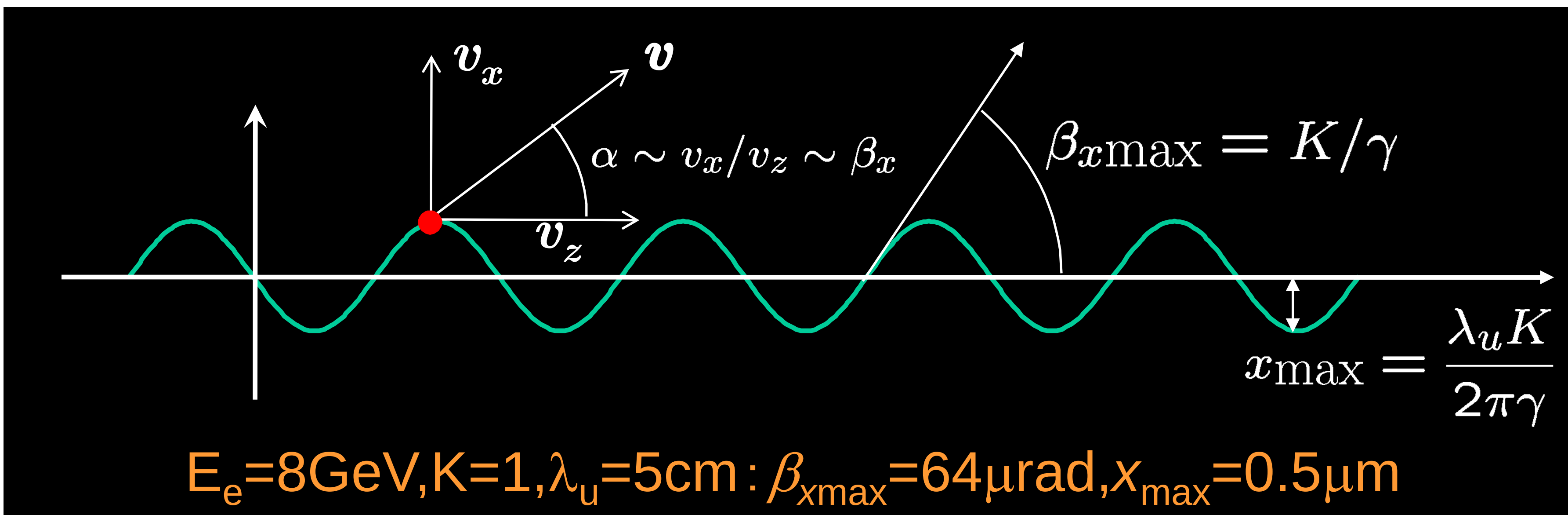
# Trajectory in an Ideal ID

$$\begin{cases} B_x(z) = 0 \\ B_y(z) \sim B_0 \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \quad \begin{cases} \beta_y = 0 \\ \beta_x = \frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \quad \begin{cases} y = 0 \\ x = \frac{\lambda_u K}{2\pi\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases}$$

magnetic field → velocity → position

$$K = \frac{eB_0\lambda_u}{2\pi mc} = 93.37 B_0(\mathrm{T})\lambda_u(\mathrm{cm})$$

K value, Deflection parameter



# Effects due to the ID Magnetic Field

transverse velocity

$$\beta_x = \frac{K}{\gamma}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

longitudinal velocity

$$\beta_z = \sqrt{\beta^2 - \beta_x^2}$$

$$= 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)$$

$\bar{\beta}_z$ :average velocity — oscillating term

ID field induces:

- transverse(x) oscillation
- longitudinal (z) oscillation
- effective deceleration($\Delta\beta_z = K^2/4\gamma^2$)

# General Form of Time Squeezing

$$\frac{d\tau}{dt} = 1 - \boldsymbol{\beta} \cdot \boldsymbol{n}$$

$$\begin{aligned} \beta_z &= \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \\ &\sim 1 - (\gamma^{-2} + \beta_x^2 + \beta_y^2)/2 \\ n_z &\sim 1 - (\theta_x^2 + \theta_y^2)/2 \end{aligned}$$

$$= \frac{1}{2\gamma^2} + (\theta_x - \beta_x)^2 + (\theta_y - \beta_y)^2$$

Time squeezing takes place most significantly when the direction of the electron motion coincides with that of observation ($\boldsymbol{\beta} = \boldsymbol{\theta}$).

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- Qualitative Description of Undulator Radiation

# Wiggler Radiation

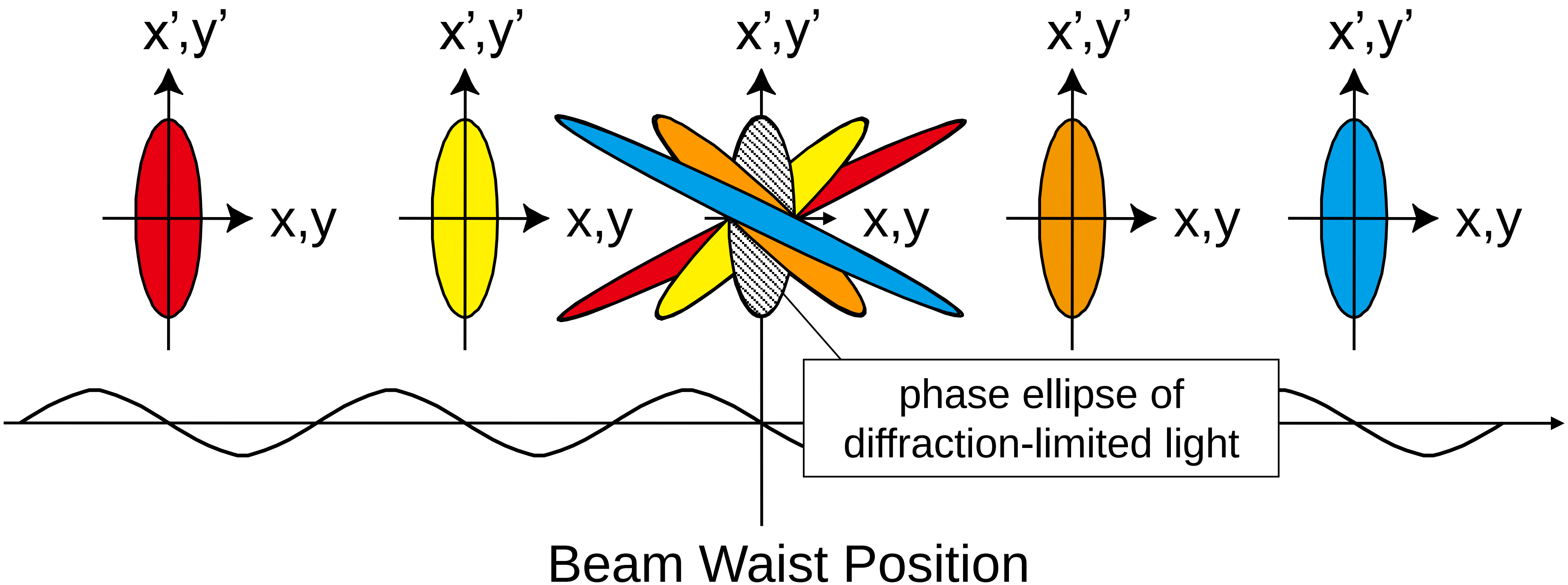
- Wiggler radiation (WR) is regarded as incoherent sum of SR at each position.
  - Summation as photons in the framework of geometrical optics.

Flux: $F_W \sim 2NF_{BM}$

Emittance: $\sigma_{x',y'} \times \sigma_{x,y} \gg \lambda/4\pi$

Brilliance: $B_W \ll 2NB_{BM}$

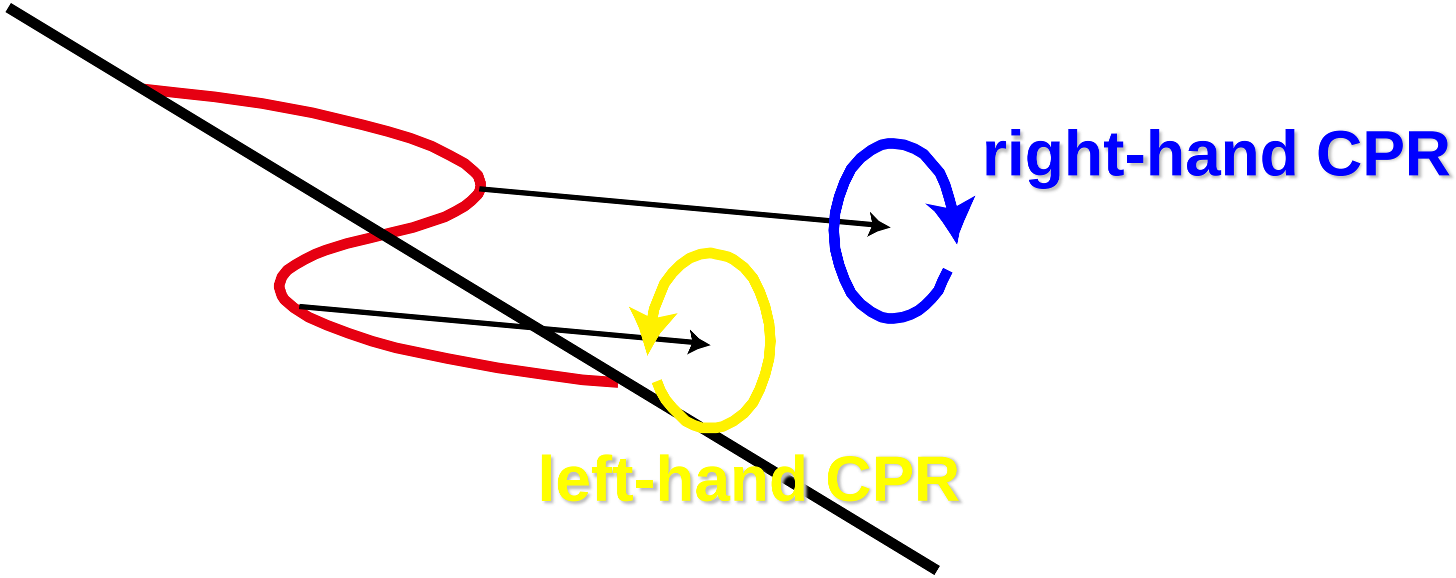
# Photon Distribution in Phase Space




- Larger *N* results in larger area of photon distribution in the phase space, i.e., larger emittance.
- *B* does not linearly depend on *N*

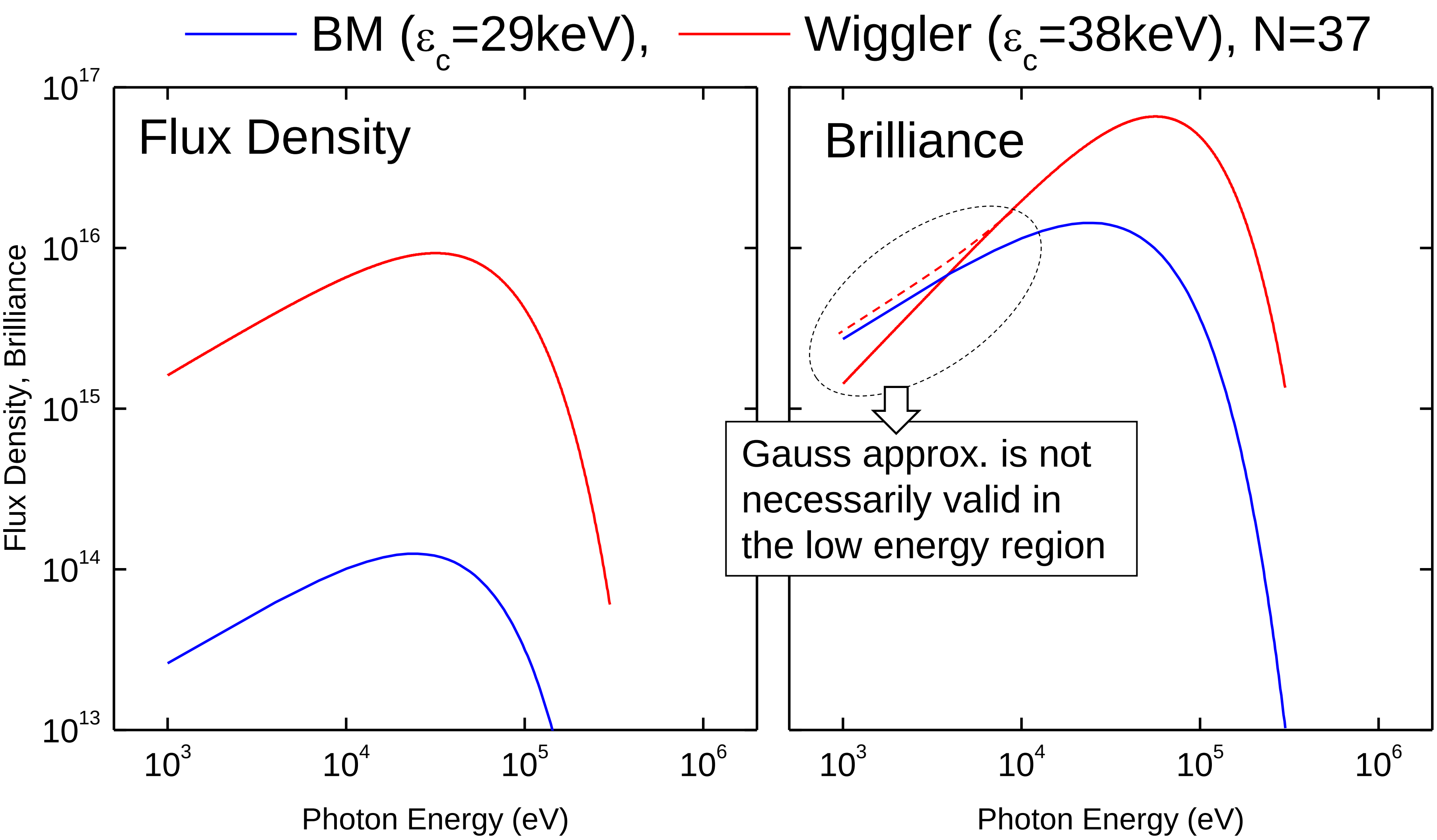
# Polarization

- No circular polarized radiation (CPR) is observed unlike the BM radiation even off axis, due to cancellation of CPR components.




- EMPW is a special wiggler to utilize CPR by introducing a vertical motion.

# Comparison with BM Radiation



## Characteristics of SR (2)

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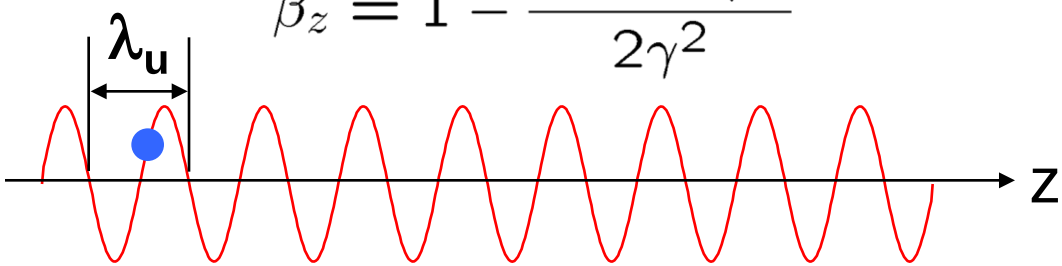
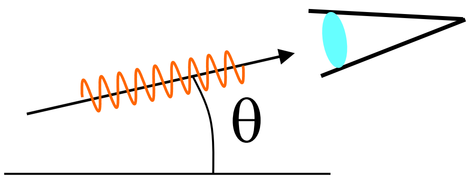
# Fundamental Wavelength

$$\bar{\beta}_z = 1 - \frac{1 + K^2/2}{2\gamma^2}$$



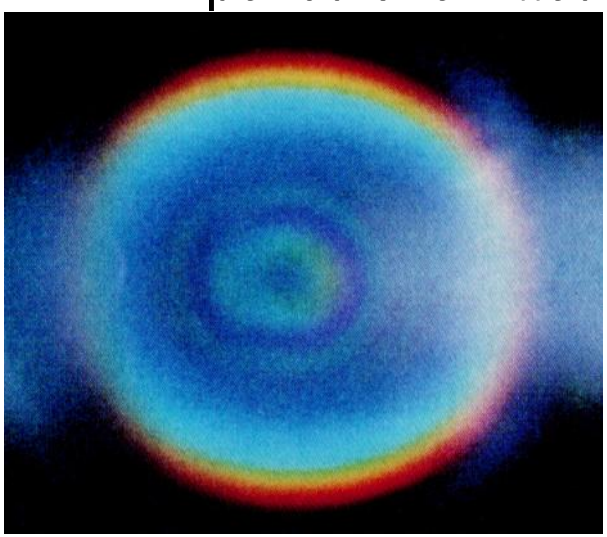

$$T = \lambda_u / v_z = \lambda_u / c$$

period of electron motion
= period of emitted light

time squeezing

$$T' = T(1 - \bar{\beta}_z \cos\theta)$$

period of observed light



H. Kitamura et al.,
J. Appl. Phys. 21 (1982) 1728

## Fundamental Wavelength $\lambda_1$

$$\lambda_1 = \lambda_u(1 - \bar{\beta}_z \cos\theta) = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2 + K^2/2)$$

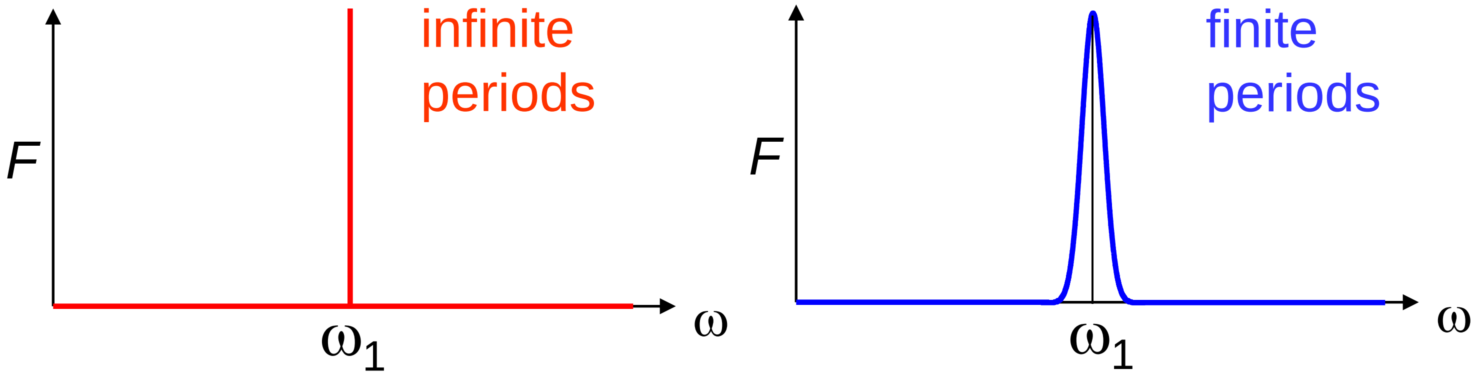
$$\omega_1 = 2\pi c/\lambda_1$$

# UR with Infinite Periods

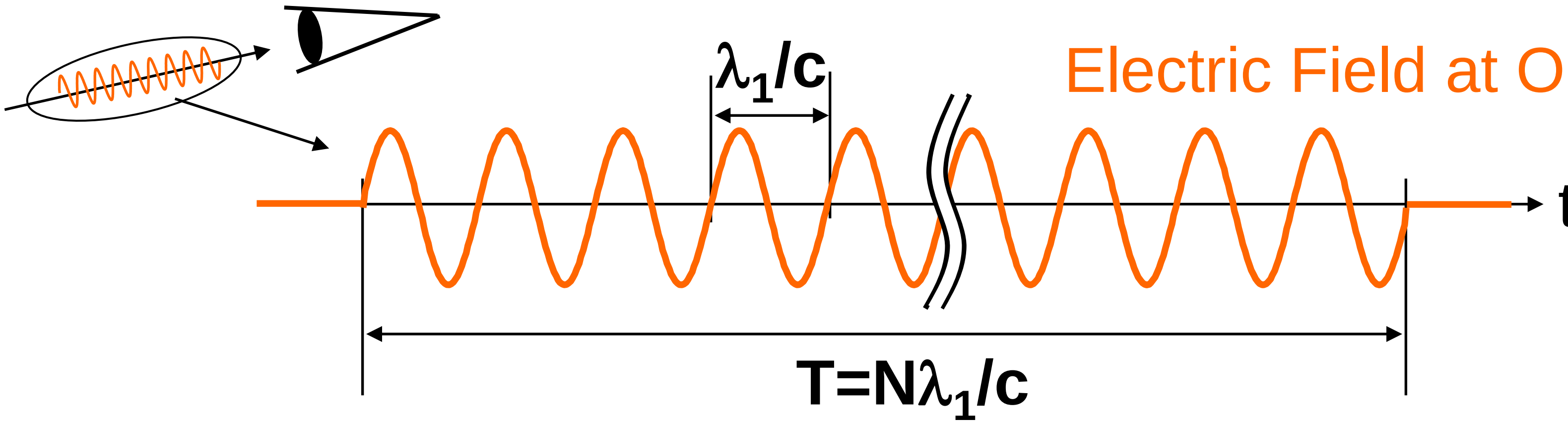
- If the undulator length is infinite, the pulse duration is infinitely long, and thus the radiation is completely monochromatic with line spectrum.

$$\frac{d^2F}{dx'dy'} \propto \delta(\omega - \omega_1) = \delta\left(\omega - \frac{4\pi c\gamma^2/\lambda_u}{1 + K^2/2 + \gamma^2\theta^2}\right)$$

- In practice, the undulator length is finite, so the line spectrum is broadened.



# Effects due to Finite Periods




$$E(t) = \begin{cases} E_0 \sin\omega_1 t & ; -T/2 \le t \le T/2 \\ 0 & ; t < -T/2, T/2 < t \end{cases}, \quad \omega_1 = 2\pi c/\lambda_1$$

Fourier Transform

$$\frac{d^2F}{dx'dy'} \propto |\tilde{E}(\omega)|^2 \propto \mathrm{sinc}^2\left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)}\right]$$

Square of “sinc” function dominates the UR

# Brief Note on UR Formulae

- In the previous derivations of UR spectral function, no knowledge on electro-dynamics is required.
- In practice, $E_0$ is a complicated function of $\theta$ and $K$, and needs to be calculated by Fourier transforming the electric field derived from the Lienard-Wiecherd potential.
- However, the simple derivation gives us a clear understanding on UR properties.

# Energy and Angular Profile of UR

$$\frac{d^2 F(\omega,\theta)}{d\Omega d\omega/\omega} = F_0 \mathrm{sinc}^2\left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)}\right]$$

$=$

**Energy Profile at $\theta = 0$**

$$F_0 \mathrm{sinc}^2(N\pi\varepsilon)$$
$$;\varepsilon = [\omega - \omega_1(0)]/\omega_1(0)$$

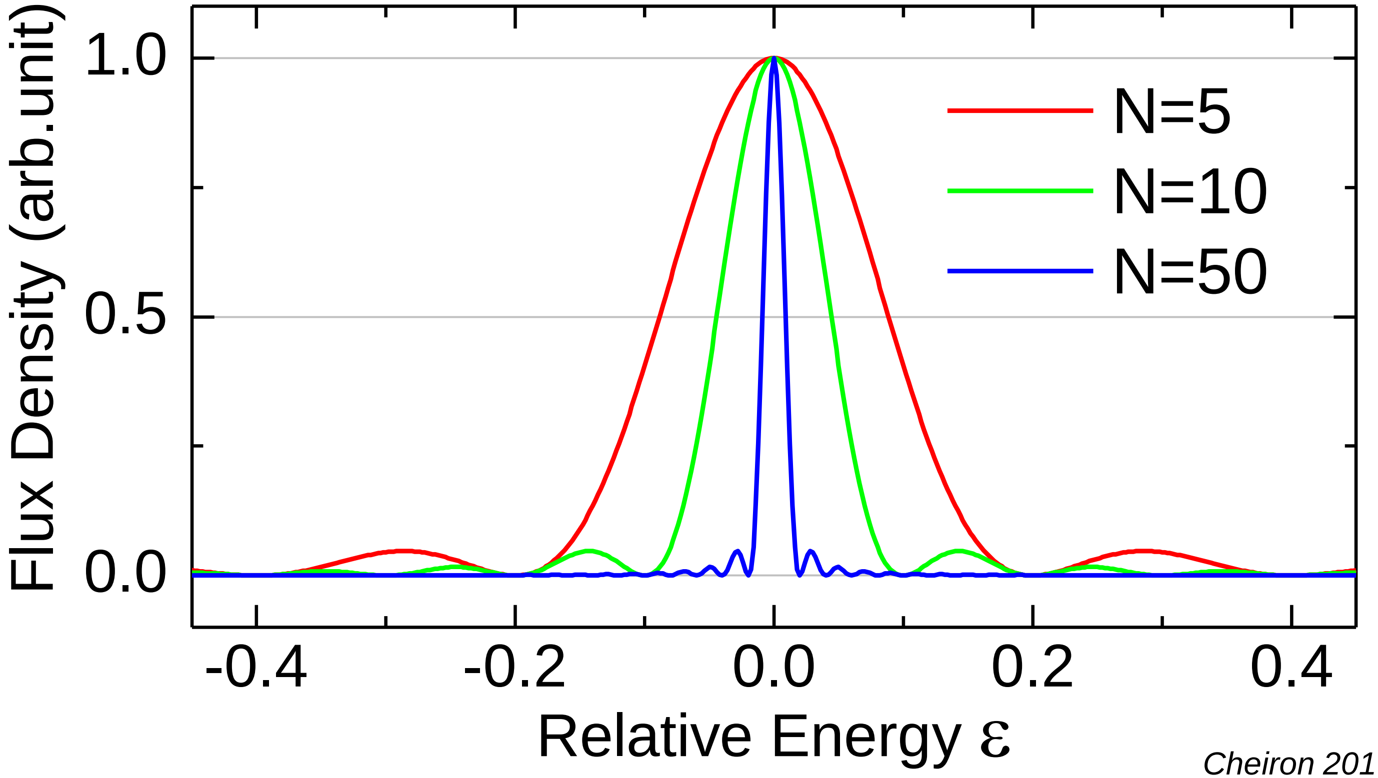
**Angular Profile at $\omega=\alpha\omega_1(0)$**

$$F_0 \mathrm{sinc}^2[N\pi(\alpha\Theta^2 + \alpha - 1)]$$
$$;\Theta = \gamma\theta/\sqrt{1+K^2/2}$$

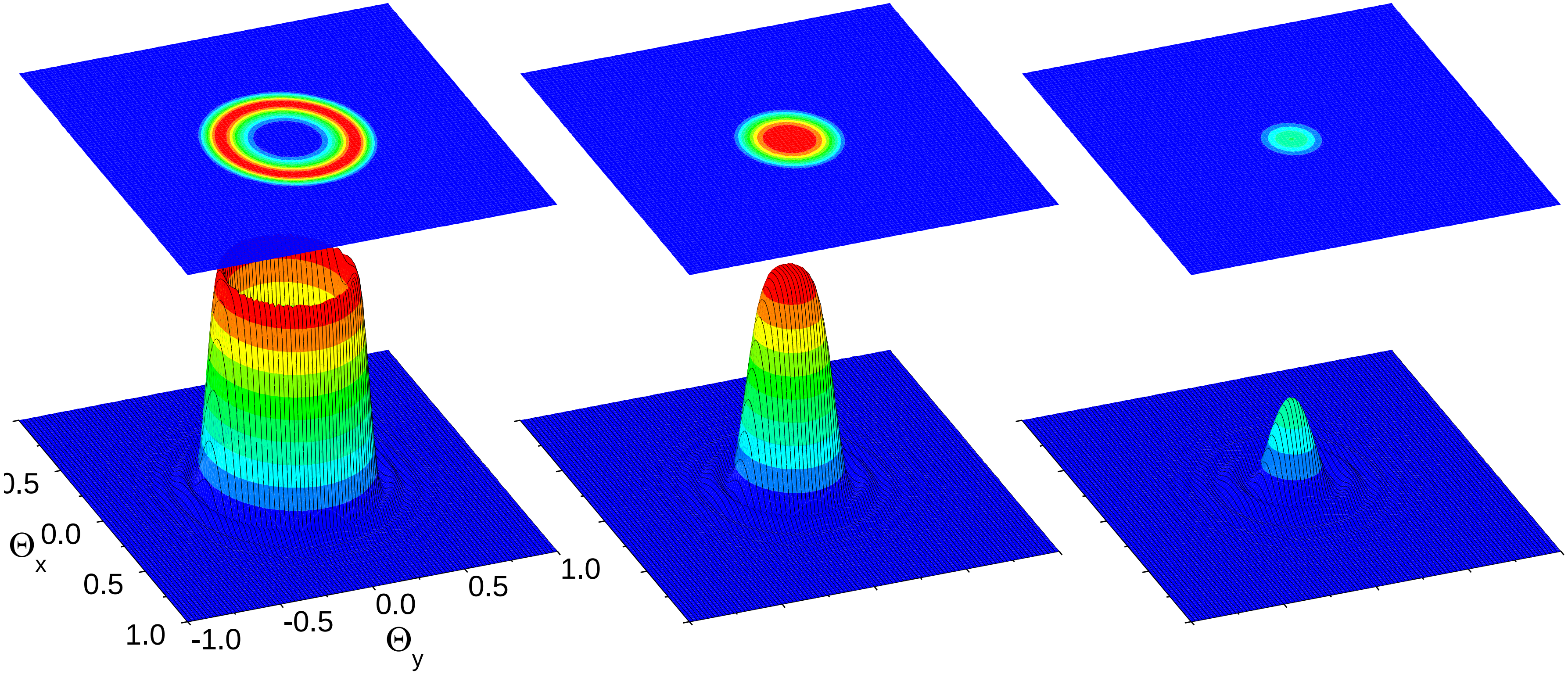
# Energy Profile: Example

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2(N\pi\varepsilon); \quad \mathrm{sinc}^2(2.783) \sim 1/2$$

$$\Rightarrow \quad \left.\frac{\Delta\omega}{\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{N}$$



# Angular Profile: Example

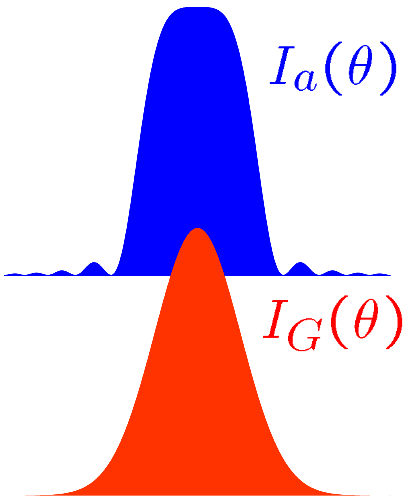



| $\omega=0.9\omega_1(0)$ | $\omega=\omega_1(0)$ | $\omega=1.05\omega_1(0)$ |
|---|---|---|
| lower energy | fundamental energy | higher energy |

# Angular Divergence and Beam Size

Angular Profile at $\omega=\omega_1(0)$

$$I_a(\theta) = F_0 \mathrm{sinc}^2\left[\frac{\pi N(\gamma\theta)^2}{1+K^2/2}\right]$$




approximation

Gaussian Profile with $\sigma_{r'}$

$$I_G(\theta) = F_0 \exp(-\theta^2/2\sigma_{r'}^2)$$

$$\sigma_{r'} = \sqrt{\frac{1+K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

Angular Divergence of UR ($L=N\lambda_u$)

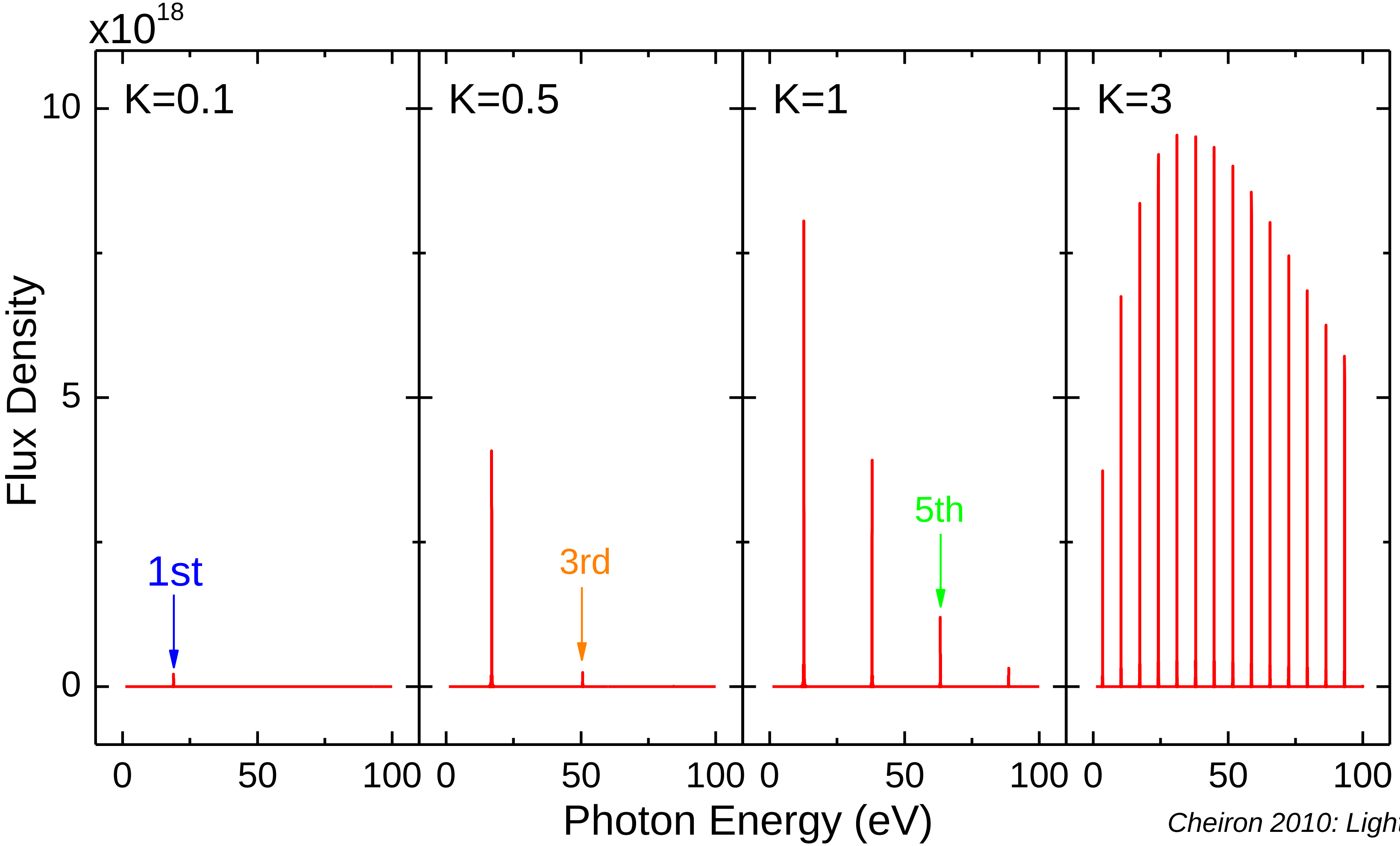
Diffraction Limit (UR is Spatially Coherent)

$$\sigma_r = \frac{\lambda_1}{4\pi\sigma_{r'}} = \frac{\sqrt{\lambda_1 L}}{4\pi}$$

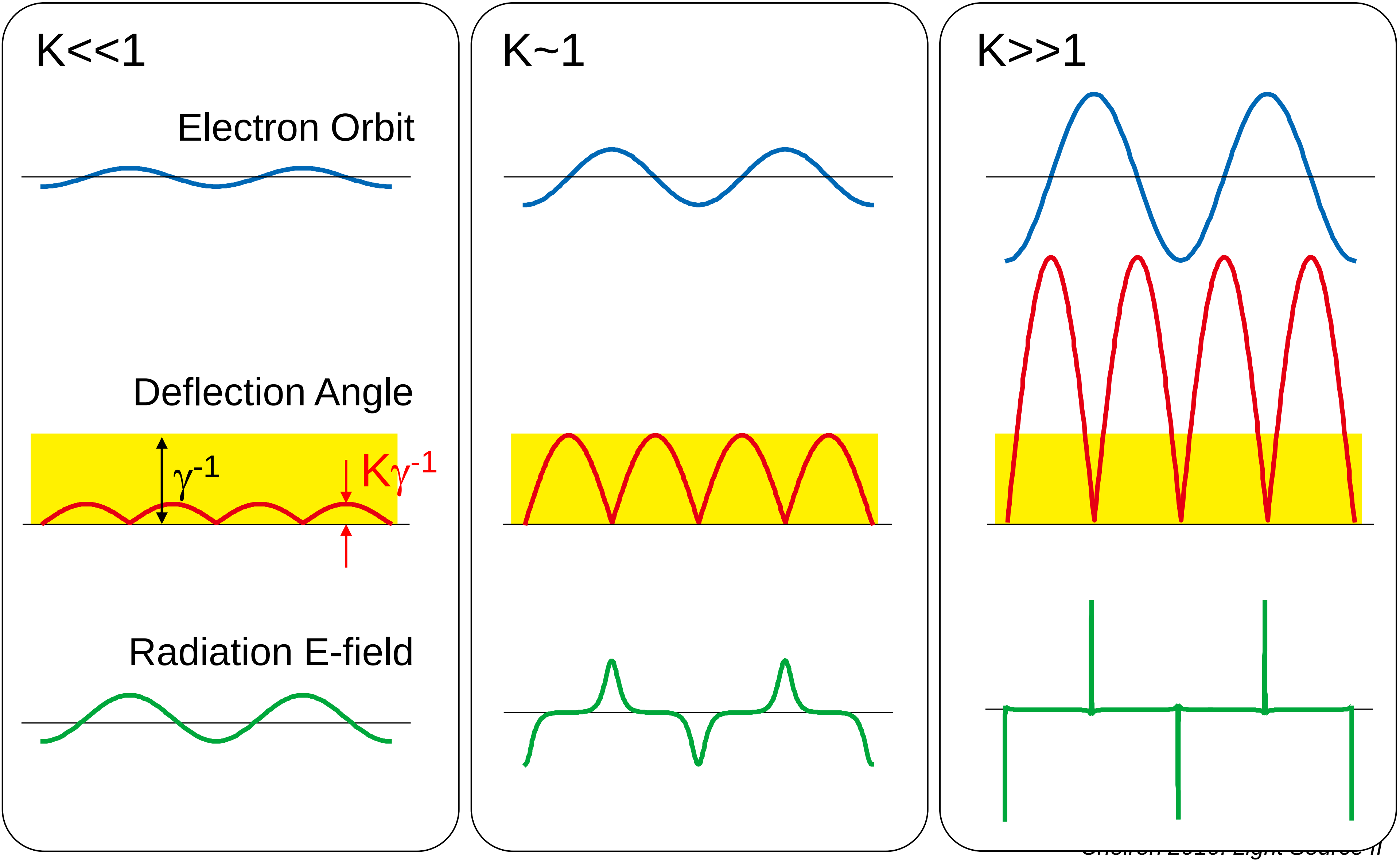
Beam Size of UR

# Higher Harmonics

- In addition to $\omega_1$, photons with the energy at $n\omega_1$ is also observed, where $n$ is an integer.



# Mechanisms of Higher Harmonics



# Optical Properties of Higher Harmonics

For the n-th harmonic radiation,

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2\left[\pi n N \frac{\omega - n\omega_1(\theta)}{n\omega_1(\theta)}\right]$$

$$\left.\frac{\Delta\omega}{n\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{nN} \quad \text{band width}$$

$$\sigma_{r'n} = \sqrt{\frac{1+K^2/2}{4nN\gamma^2}} = \sqrt{\frac{\lambda_1/n}{2L}} \quad \text{angular divergence}$$

$$\sigma_{rn} = \frac{\lambda_1/n}{4\pi\sigma_{r'n}} = \frac{\sqrt{L\lambda_1/n}}{4\pi} \quad \text{beam size}$$

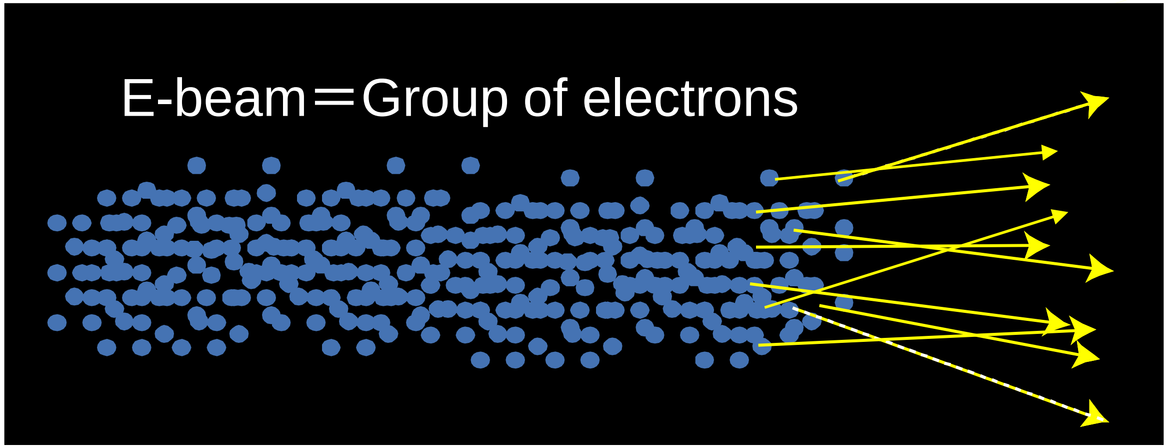
# Practical Knowledge on Undulator Radiation

# "Practical Knowledge" on UR

- The undulator is a "double-edged sword"!
  - Much higher brilliance is available than the BM and wiggler, *if appropriately used*.
  - High heat load on optical elements
  - Quasi-monochromatic and small angular spread imposes an accurate adjustment of BL components.
- Practical knowledge for the utilization of UR
  - Effects due to the electron beam quality
  - Simple evaluation of optical properties
  - Heat load reduction
  - …….

# Electron Beam Quality (1)

- The property of UR from a single electron is similar to a laser
  - Small Size & Angular Divergence, Narrow Bandwidth
- In practice, UR in the beamline is emitted by the beam comprising a huge number of electrons.
- These electrons have different positions, angle, and energies.



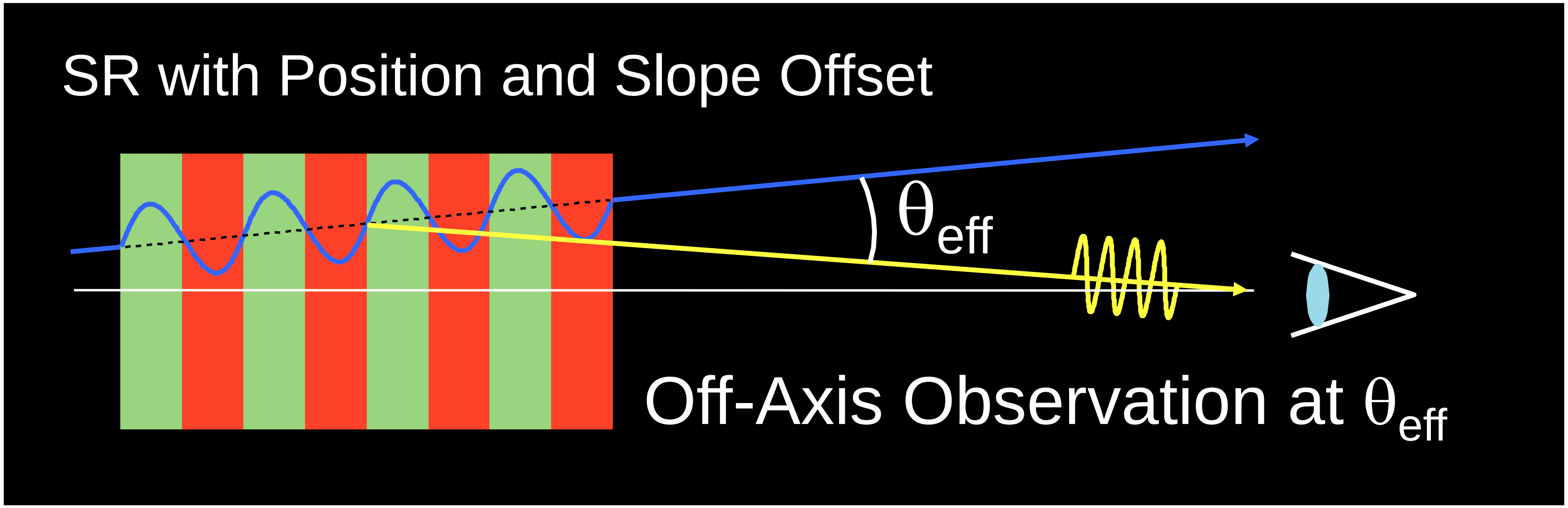

The performance of UR depends largely on the e-beam quality

# Electron Beam Quality (2)

- Electron-beam parameters related to the performance of UR
  - Beam Size $\sigma_x,\sigma_y \rightarrow$ Source Size
  - Angular Divergence $\sigma_{x'},\sigma_{y'} \rightarrow$ Directivity
    - The minimum value of the product $\sigma_x\sigma_x$ and $\sigma_y\sigma_y$ are called the electron beam emittances in the x and y directions.
  - Energy Spread $\sigma_E/E \rightarrow$ Monochromaticity

# Effects due to Finite Emittance (1)

- Effects due to Finite Emittance of the Electron Beam
  - Injection to the undulator with angular and positional offset

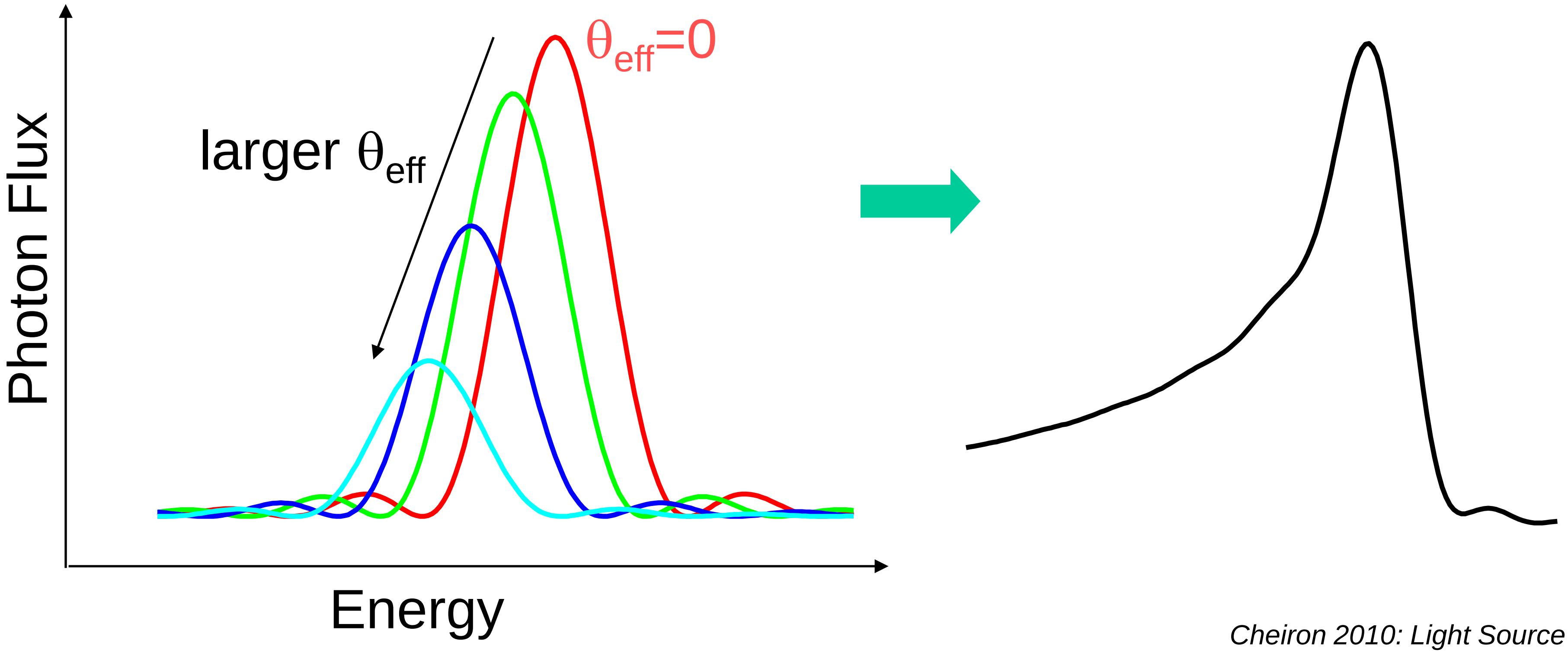


# Effects due to Finite Emittance (2)

Off-axis observation at $\theta_{eff}$

→ Peak shift to lower energy

$$\omega_1(\theta) = \frac{4\pi c\gamma^2/\lambda_u}{1 + \boxed{\gamma^2\theta^2} + K^2/2}$$

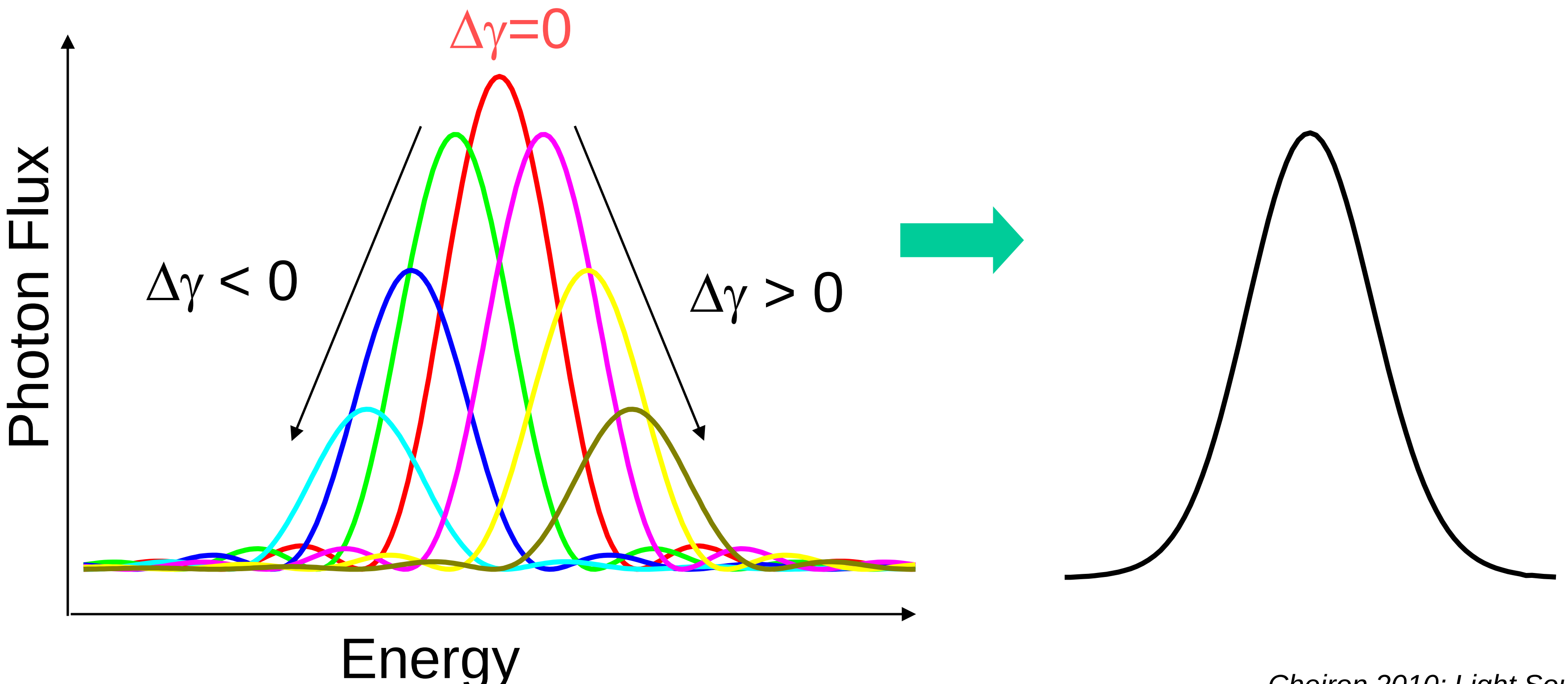


# Effects due to the Energy Spread

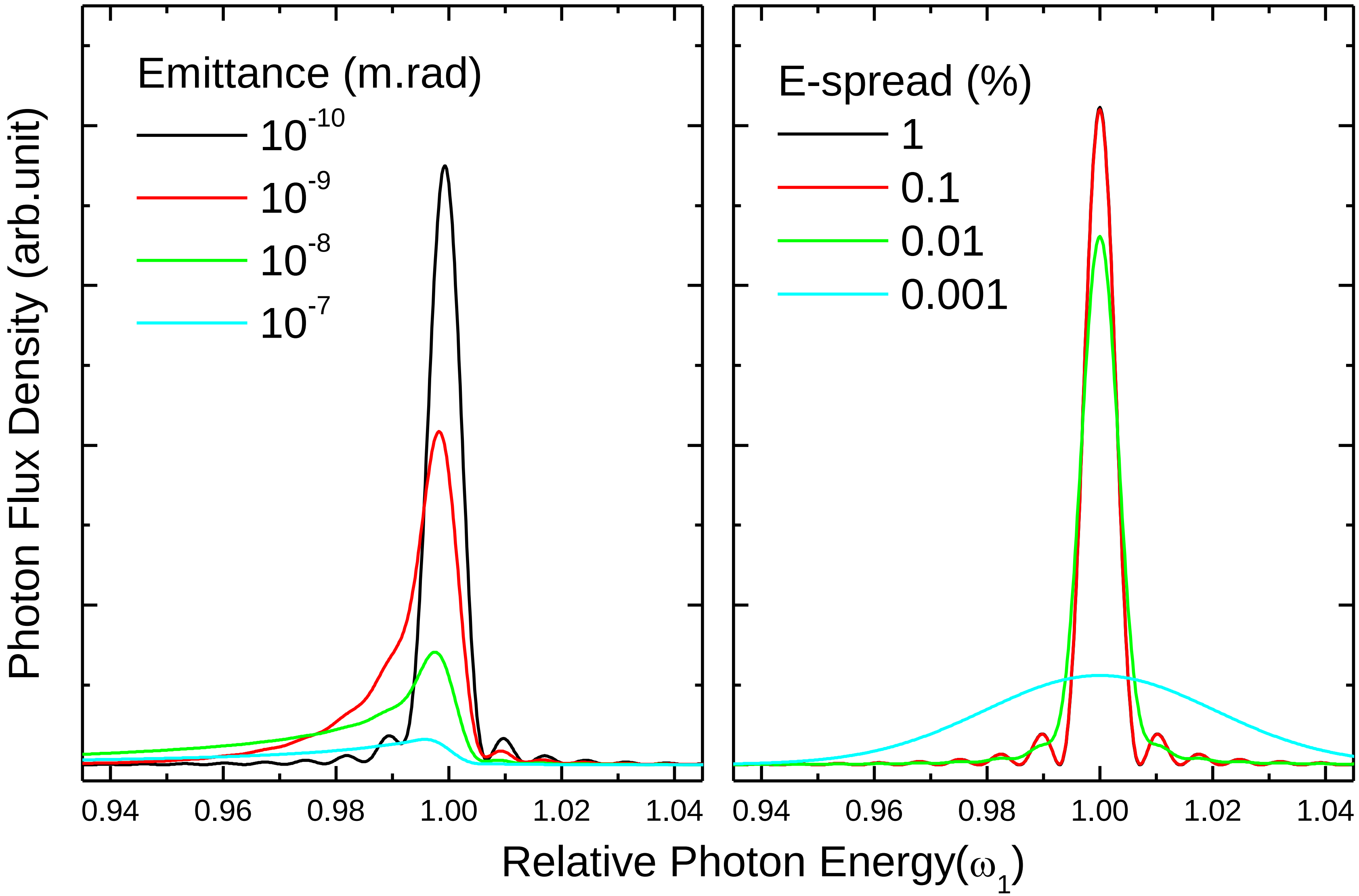
Electron with an offset of $\Delta\gamma$

Energy shift of $\omega_1$

$$\omega_1(\gamma) = \frac{4\pi c\gamma^2/\lambda_u}{1+K^2/2}$$



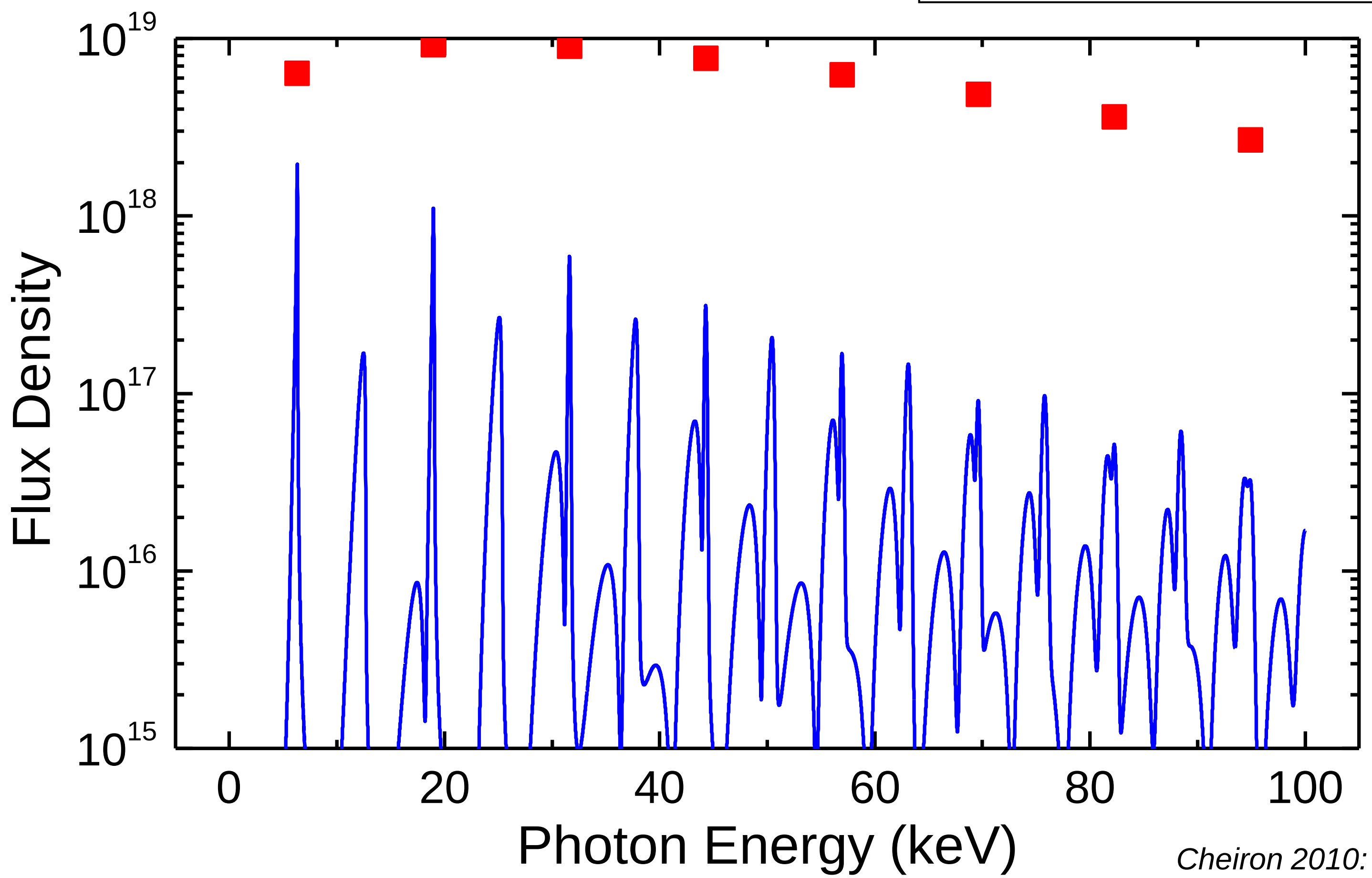
# Examples


Photon Flux Density (arb.unit)
Emittance (m.rad)
10-10
10-9
10-8
10-7
E-spread (%)
1
0.1
0.01
0.001
0.94
0.96
0.98
1.00
1.02
1.04
Relative Photon Energy(ω1)

# Effects on the Higher Harmonics

Optical Emittance of UR: $\lambda/4\pi$
Bandwidth of UR: $\sim 1/nN$

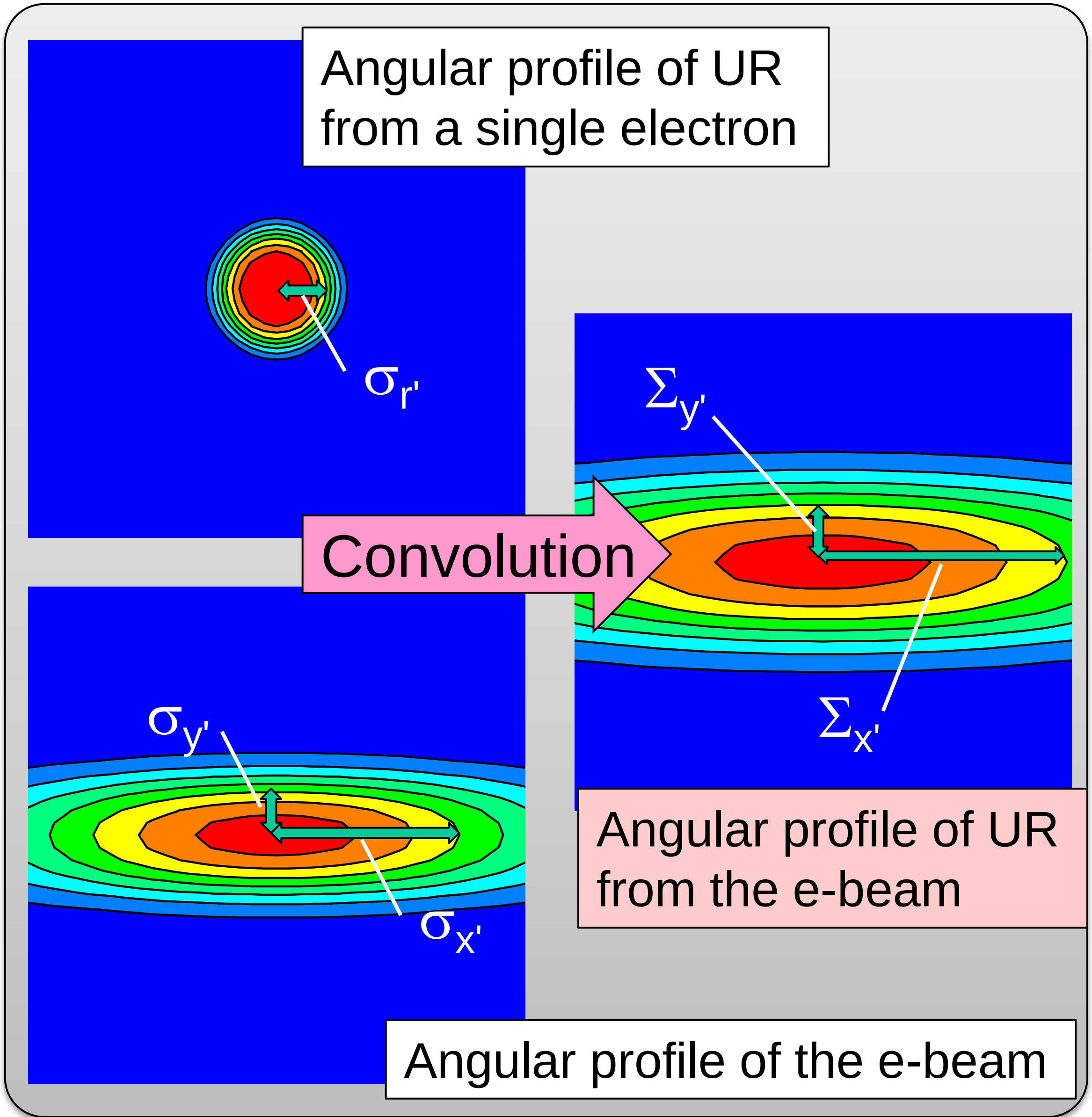
Effects due to the $e^-$ beam are larger for higher harmonics



# Evaluation of Optical Properties

- Optical properties of UR from the e-beam:
  - Calculate the UR from a single electron based on the theory of electrodynamics.
  - Integrate over the electrons in the beam (convolution)
    - Requires complicated numerical computation with a large number of parameters
    - Dedicated computer software (SPECTRA, SRW,…) is available
- Easy evaluation with Gauss approxmation
  - Source size, angular divergence
  - Flux density, brilliance

# Effective Size & Divergence




By Gauss approx. and convolution theorem,

$$\Sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{x',y'}^2}$$

Effective Angular Div.

$$\Sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{x,y}^2}$$

Effective Source Size

# Effective Flux Density and Brilliance

$$\int_{-\infty}^{\infty} G\,\mathrm{exp}(-x^2/2\sigma^2)dx = G \times \sqrt{2\pi}\sigma$$

Effective width of a Gauss function is $\sqrt{2\pi}\,\sigma$

**Total Flux** $F = \left.\frac{d^2F}{dx'dy'}\right|_0 \times 2\pi\sigma_{r'}^2$

on-axis flux density with zero-emittance beam

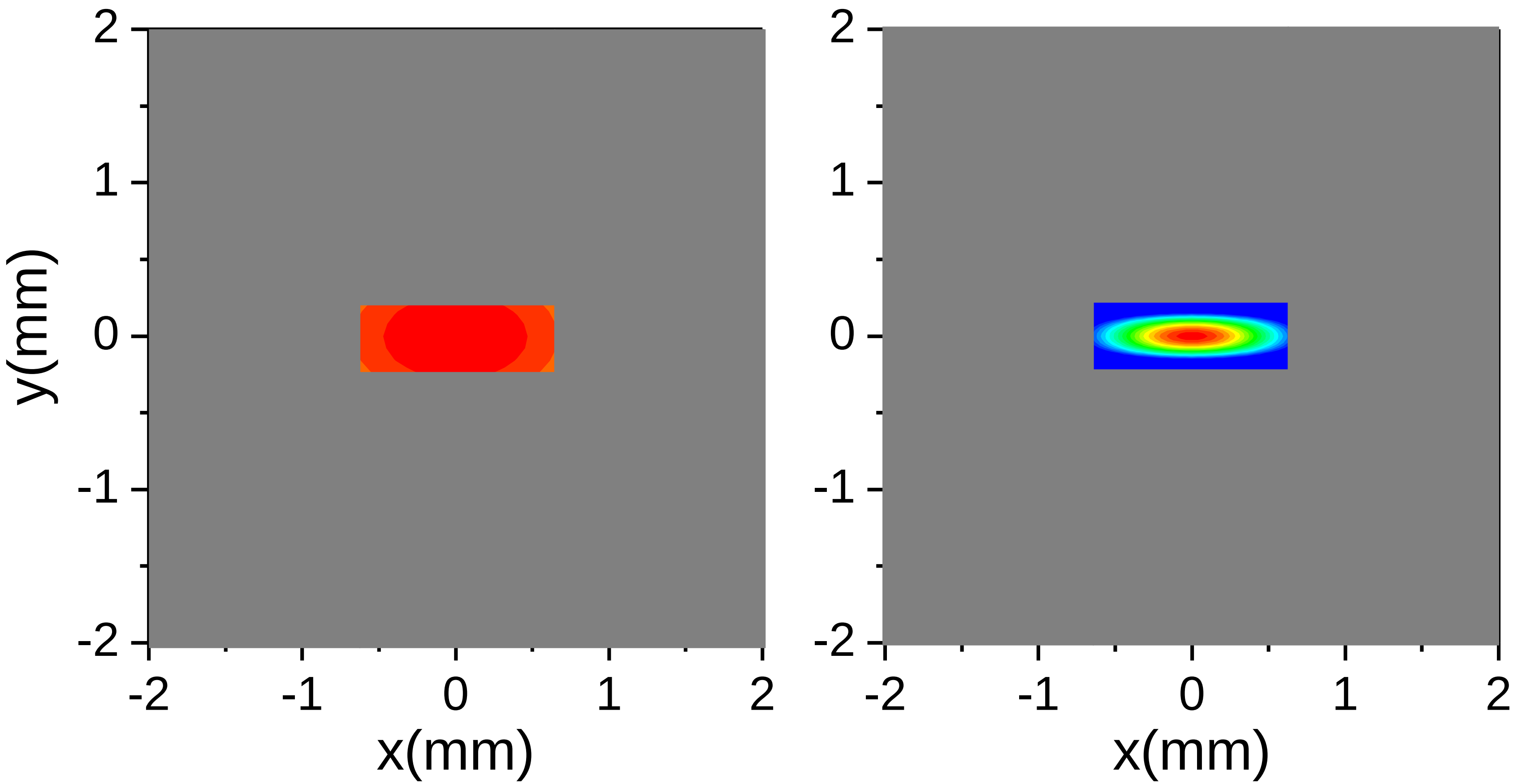
**Effective Flux Density** $\left.\frac{d^2F}{dx'dy'}\right|_e = \frac{F}{2\pi\sigma_{x'}\sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{\sigma_{x'}\sigma_{y'}}$

**Effective Brilliance** $B_e = \frac{F}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{2\pi\sigma_x\sigma_{x'}\sigma_y\sigma_{y'}}$

# Heat Load on Optical Elements

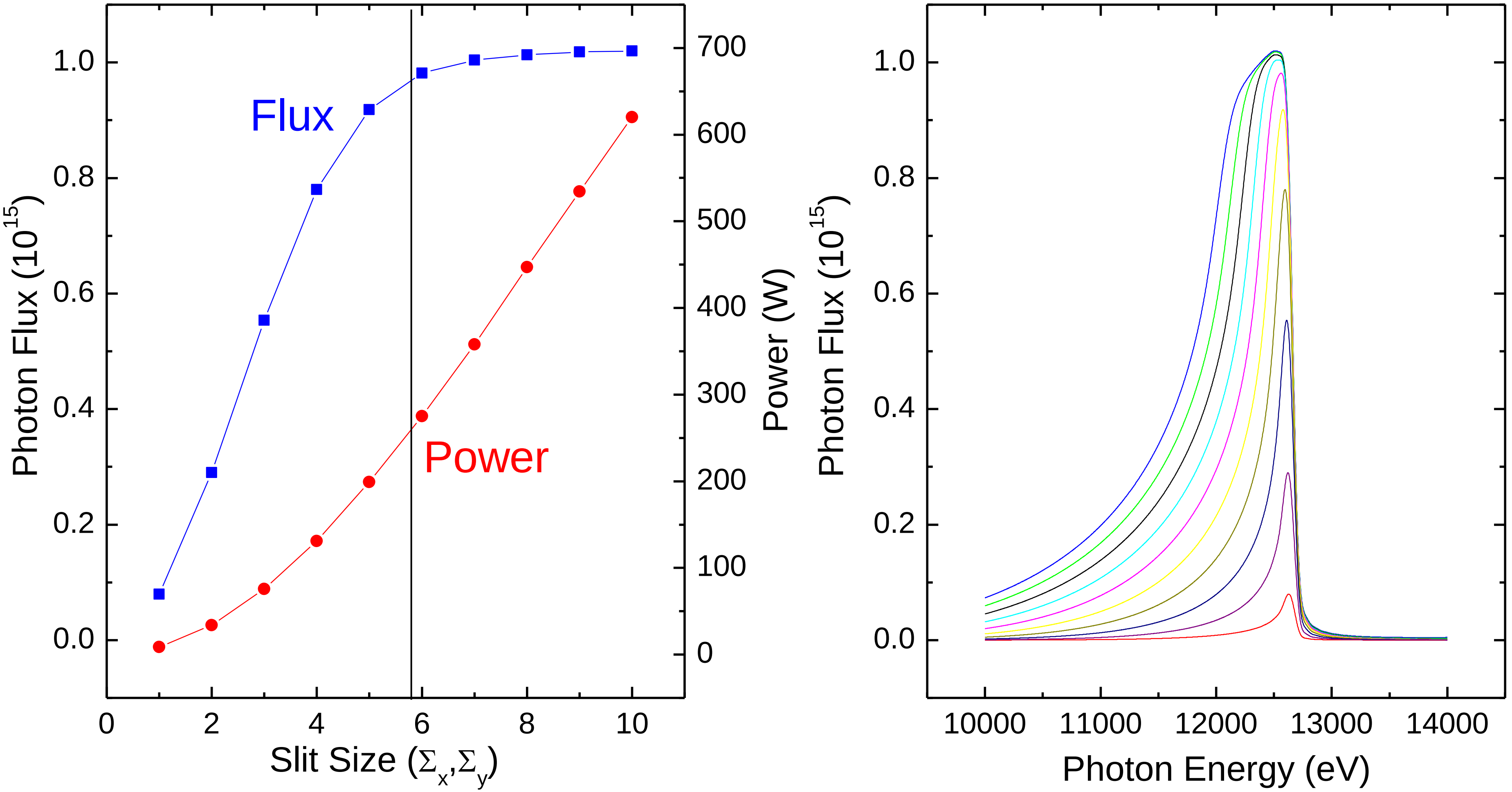
- SR emitted from the light source is processed by several optical elements before irradiation to the sample, such as the focusing mirror, monochromator.
- These elements can be easily damaged by the heat load brought by the SR.
- It is thus important to reduce the heat load as much as possible without sacrificing the flux by means of the XY slit at the front-end section.

# Spatial Profile of Power and Flux




The power profile is much broader than the flux. Extraction of SR with an appropriate slit significantly reduces the heat load.

# Optimum Slit Size?

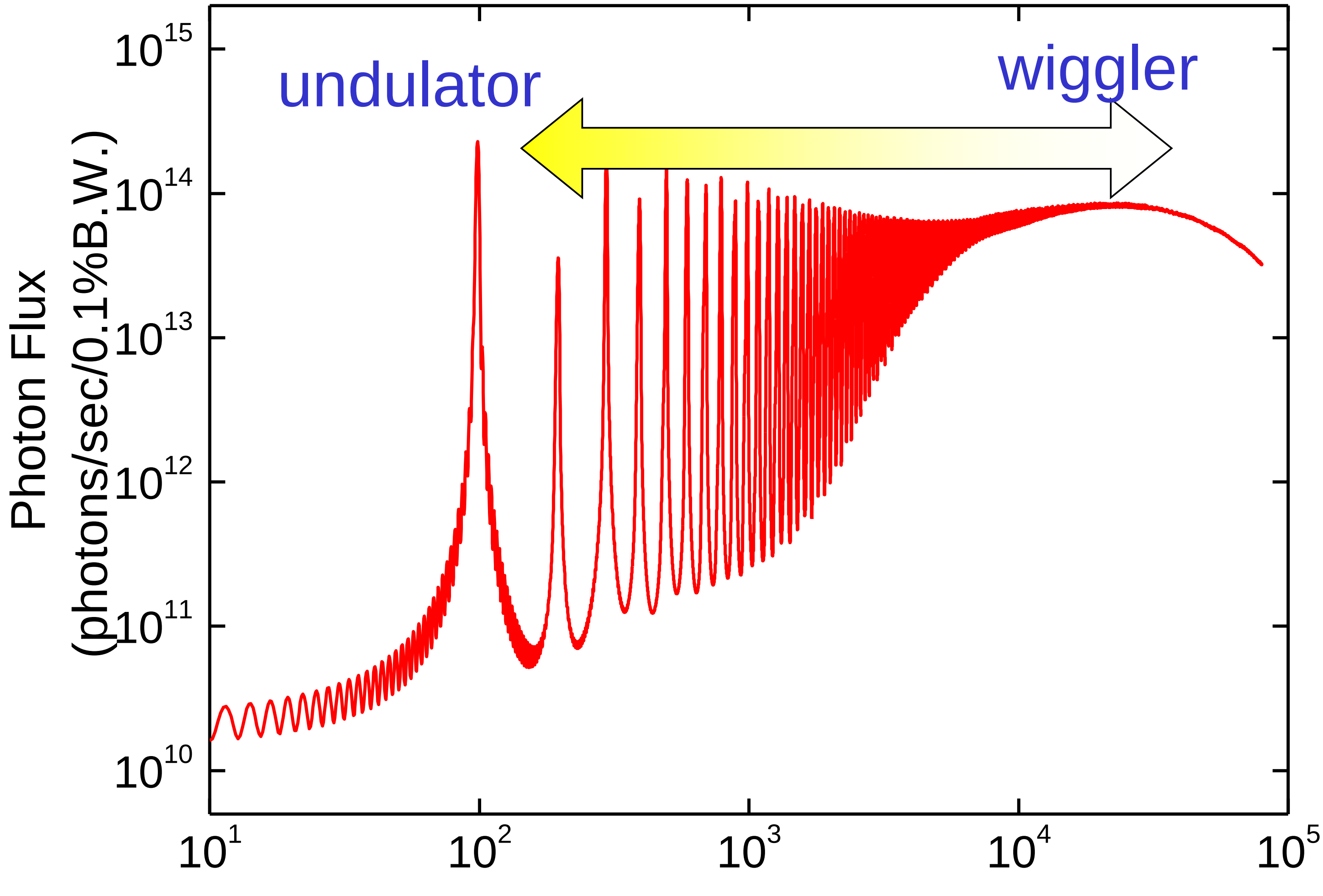



Too wide aperture results in broader bandwidth and unwanted heat load.

# Wiggler? Undulator? (1)

- Wigglers are identical to undulator from the point of view of magnetic circuit.
- It is generally said that the K value distinguishes between the two, however, this is not exactly correct.
- What we should take care is the region of photon energy to be utilized for application.

# Wiggler? Undulator? (2)



# Undulator Radiation Gallery

- For quantitative evaluation of SR, a computer code "SPECTRA", which has been developed and maintained in SPring-8 is available.
- SPECTRA also offers a function to "visualize" the computation results for further understanding of SR.
  - brilliance curve & spectrum
  - on- and off-peak angular profiles of flux
  - on- and off-axis spectra
  - effects of opening the slit aperture
  - undulator-to-wiggler transition

# Other Topics Not Addressed

- Quantitative descriptions of SR
- Light sources for circular polarization and schemes for fast helicity switching
  - helical undulator & elliptic wiggler
  - chicanes&choppers, kicker magnets
- Effects on the electron beam
  - natural focusing
  - beam-axis fluctuation due to COD variation
- R&Ds toward shorter magnetic period
  - superconducting undulators
  - cryogenic permanent magnet undulators
- Coherent SR for intense THz light
- Undulators for SASE-based X-ray FEL