

# Light Source I

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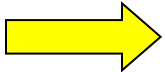
# CONTENTS

Light Source I



- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)

Light Source II



- Characteristics of SR (2)
- Practical Knowledge on SR
- Numerical Characterization of SR

# Introduction

# SR Facility and Light Source

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- SR: Definition
  - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- SR Facility
  - Accelerators to generate a high-energy electron beam
  - **Magnetic devices (SR light source) to generate intense SR**
  - Optical elements (monochromators, mirrors,..)
  - Experimental stations

# SR as a Probe for Research

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- SR has a lot of advantages over other conventional light sources
  - Highly collimated (laser-like)
  - Wavelength tunability
  - Polarization
  - .....
- However, the total radiation power does not differ significantly.



Comprehensive understanding of SR (and light source) is required for efficient experiments.

# Topics in This Lecture (1)

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- Fundamentals of Light and SR
  - Why we need SR?
  - Physical quantity of light
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet configuration
- Characteristics of SR (1)
  - Radiation from bending magnets

# Topics in This Lecture (2)

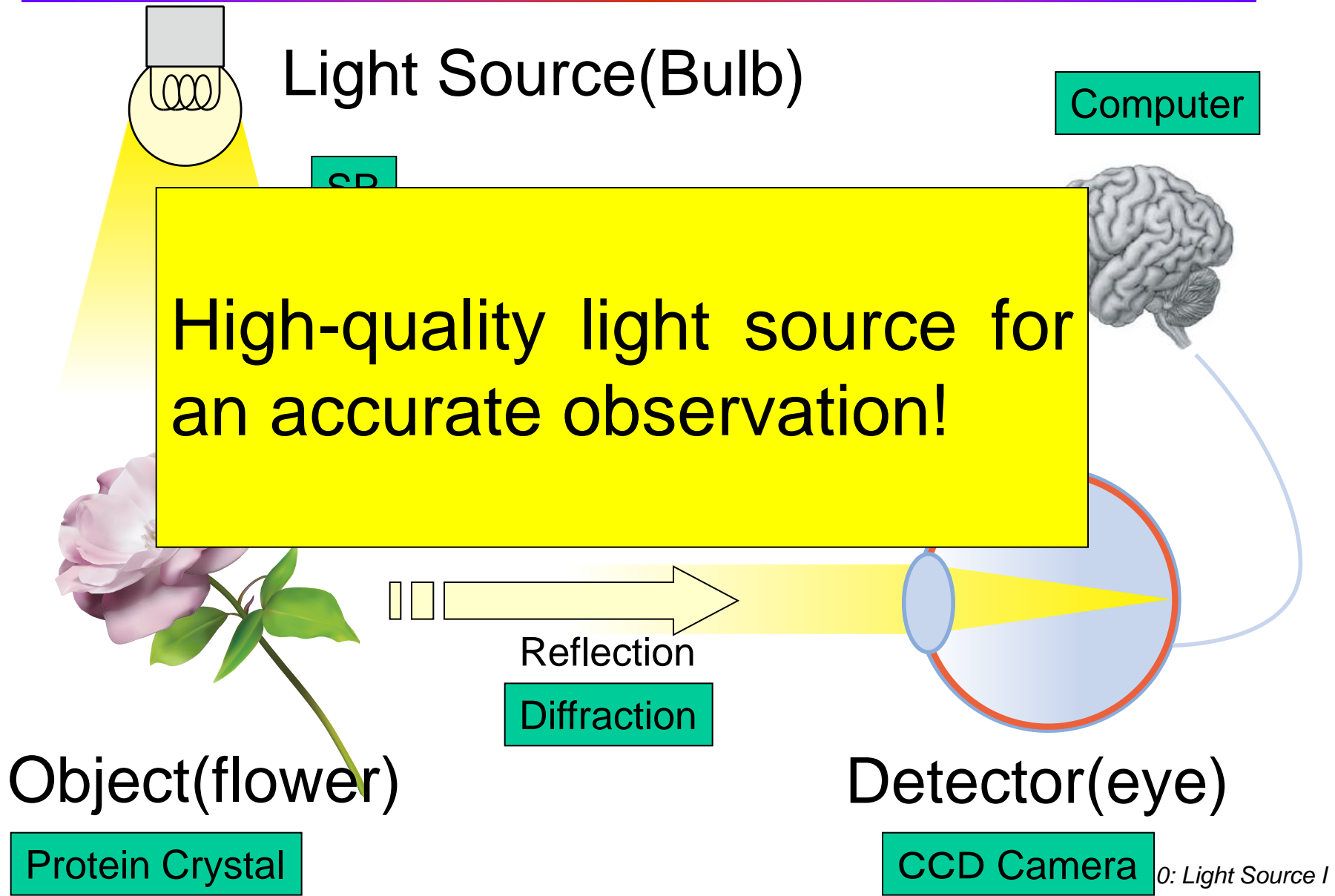
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- Characteristics of SR (2)
  - Electron Trajectory in IDs
  - Radiation from wigglers
  - Radiation from undulators
- Practical Knowledge on SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Definition of undulators and wigglers
- Numerical Characterization of SR
  - Introduction to “SPECTRA” and examples

# Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

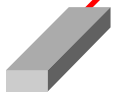
# Observation with Light



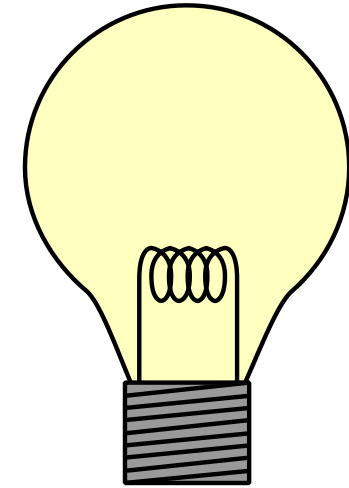
# Which Quality is Better?

Specs of SPring-8

- E = 8GeV
- I = 100mA
- L = 1500m



1mW Laser (pointer)



100W Bulb

Lighting equipment in a room:

Bulb

Pointer during a presentation:

Laser

**Depends on the Object!**

# How to Define the Quality of Light?(1)

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- The performance of the light source depends on the dimension of the object and the method to detect light.
- For observation, the photons emitted by the light source should be
  - illuminated on the object for interaction
  - recognized by the detector for analysis

Quality of Light Source:  
How efficiently the above  
conditions are satisfied?

# How to Define the Quality of Light?(2)

## Important Features of the Light Source

	Object		Related Items
	Flower	Protein	
Radiation Power	◎	○	# Emitted Photons
Source Size	×	◎	Illuminated Area
Directivity	△	◎	
Monochromaticity	△	◎	Accuracy of Analysis

**Brilliance**

# What is Brilliance?

Brilliance (photons/sec/mm<sup>2</sup>/mrad<sup>2</sup>/0.1%B.W.)

Total Power

~  $\frac{\text{Total Power}}{\text{Source Size} \times \text{Angular Divergence} \times \text{Band Width}}$

- Brilliance specifies the quality of light for observation of microscopic objects.
- The brilliance of a light source with a high total power is not necessarily high.

# Examples of Brilliance

	Bulb	Laser Pointer
Total Power (W)	100	$10^{-3}$
Angular Div. (mrad <sup>2</sup> )	$4\pi \times 10^6$	1
Source Size: (mm <sup>2</sup> )	$10^2$	1
Bandwidth: (%)	100	0.01
Brilliance (photons/sec/....)	$\sim 10^8$	$\sim 10^{16}$

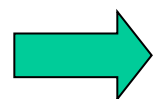
**Laser is the best light source to observe the microscopic object!**

# X ray as a Probe

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- Definition (not unique)
  - Electromagnetic wave (= light) with  $\lambda$  of 10 nm ( $10^{-8}$  m)  $\sim$  0.1 Å ( $10^{-11}$  m)
- Properties
  - High Energy/Photon
  - High Penetration (Roentgen etc..)
- Application to Microscopic Objects
  - X-ray Diffraction
  - Fluorescent X-ray Analysis

• No Practical Lasers!!



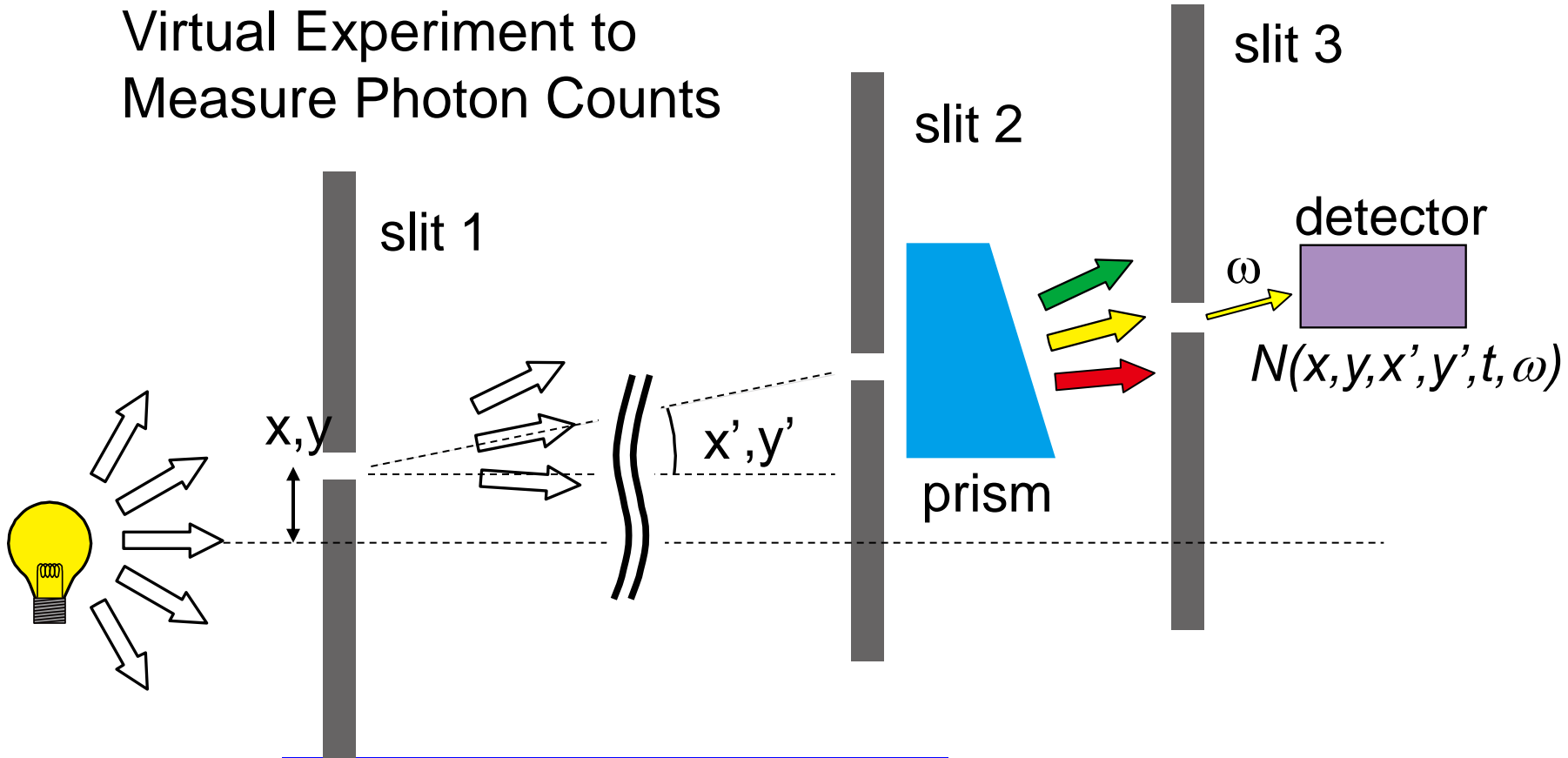
**Synchrotron Radiation(SR)**

# Fundamentals of Light and SR

- Why we need SR?
- **Physical Quantity of Light**
- Uncertainty of Light
- SR: Light from a Moving Electron

# Phase Space

Virtual Experiment to  
Measure Photon Counts



$$n = \frac{N(x, y, x', y', \omega, t)}{\Delta x \Delta y \Delta x' \Delta y' \Delta t \Delta \omega / \omega} \equiv \frac{N}{\Delta \Omega}$$

Av. Photon Density

**Volume in 6-D  
Phase Space**

# Brilliance (Brightness)

- Brilliance (photons/sec/mm<sup>2</sup>/mrad<sup>2</sup>/0.1%B.W.) is defined as the **photon density in the 6D phase space**, i.e.,

$$B = \lim_{\Delta\Omega \rightarrow 0} n = \frac{d^6 N(x, y, x', y', t, \omega)}{dx dy dx' dy' dt d\omega / \omega}$$

- In practice,  $\Delta\Omega$  can never be 0 due to uncertainty of light, thus **brilliance is not a physical quantity that can be actually measured.**

# Photon Flux and Flux Density

- Removing the 1<sup>st</sup> slit gives the angular flux density (photons/sec/mrad<sup>2</sup>/0.1%B.W), i.e.,

$$\frac{d^2 F}{dx' dy'} = \iint B dx dy$$

- Removing the 1<sup>st</sup> & 2<sup>nd</sup> slits gives the total flux (photons/sec/0.1%B.W), i.e.,

$$F = \iiint B dx dy dx' dy'$$

- Estimation of number of photons to be delivered to the sample.

# Radiation Power and Power Density

- Removing the 1<sup>st</sup> & 3<sup>rd</sup> slits gives the angular power density (W/mrad<sup>2</sup>), i.e.,

$$\frac{d^2 P}{dx' dy'} = 10^3 Q_e \hbar \int \frac{d^2 F}{dx' dy'} d\omega$$

↙ conversion from photons/sec/0.1%B.W. to W

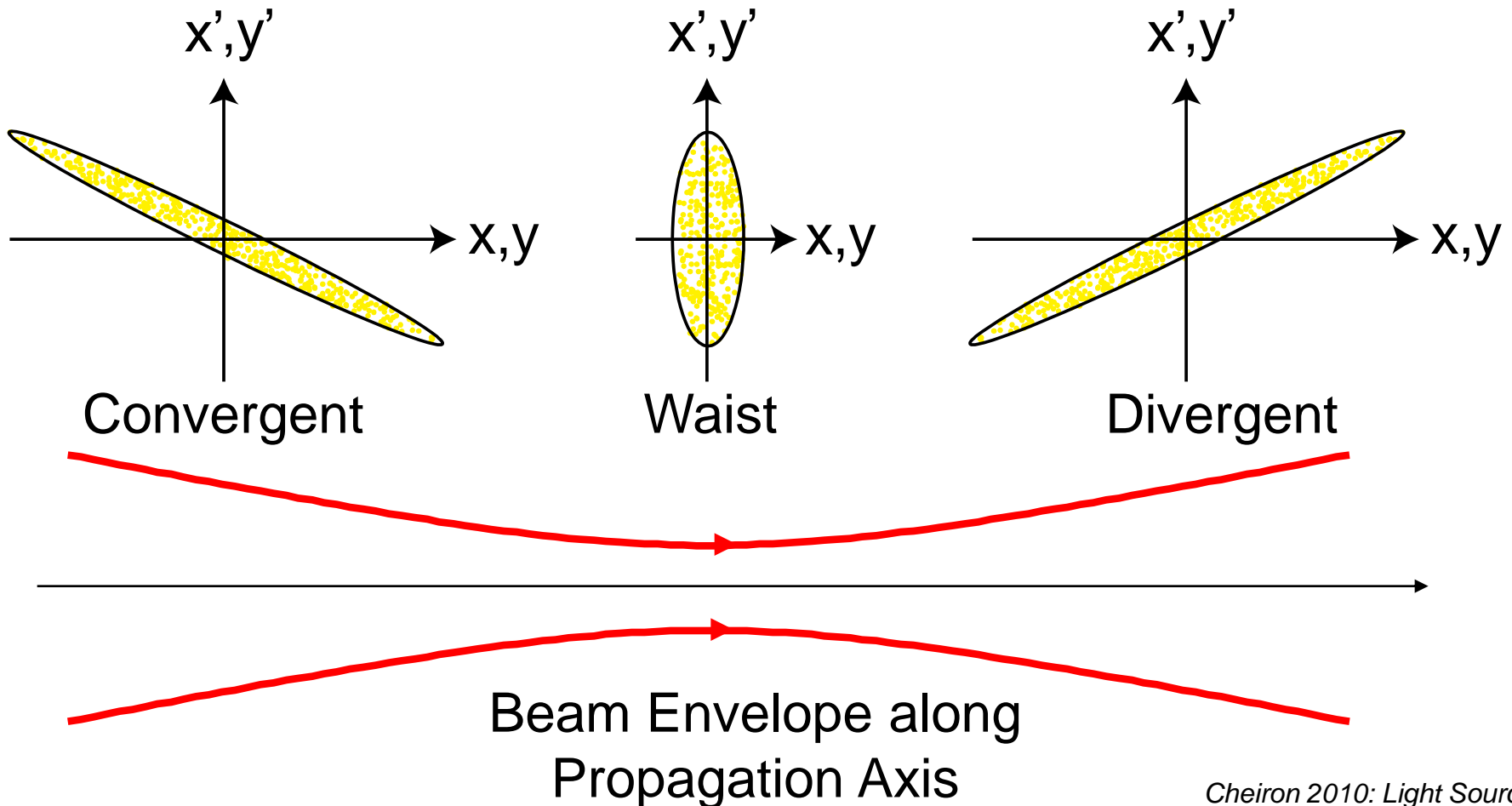
- Removing all the slits gives the total power (W), i.e.,

$$P = 10^3 Q_e \hbar \iiint \frac{d^2 F}{dx' dy'} d\omega dx' dy'$$

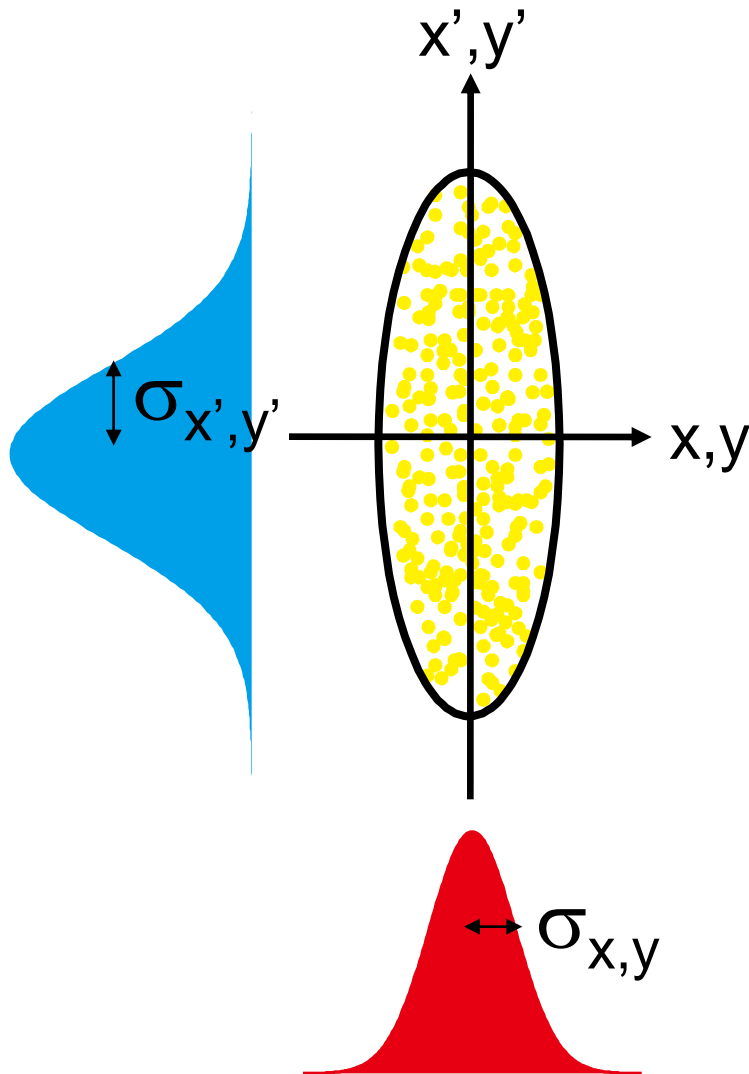
- Estimation of heat load on BL components.

# Photons in 4D Phase Space

- Photon distribution in the 4-D phase space at different longitudinal positions.



# Beam Size, Divergence, Emittance



- Beam size ( $\sigma_{x,y}$ ) is defined as the beam envelope at the beam waist position.
- Angular divergence ( $\sigma_{x',y'}$ ) is constant along the axis of propagation, as far as no optical elements are present.
- Emittance ( $\varepsilon_x, \varepsilon_y$ ) is defined as  $\sigma_{x,y} \times \sigma_{x',y'}$ , which is equal to the area of the phase ellipse divided by  $\pi$ .

# Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- **Uncertainty of Light**
- SR: Light from a Moving Electron

# Uncertainty of Light

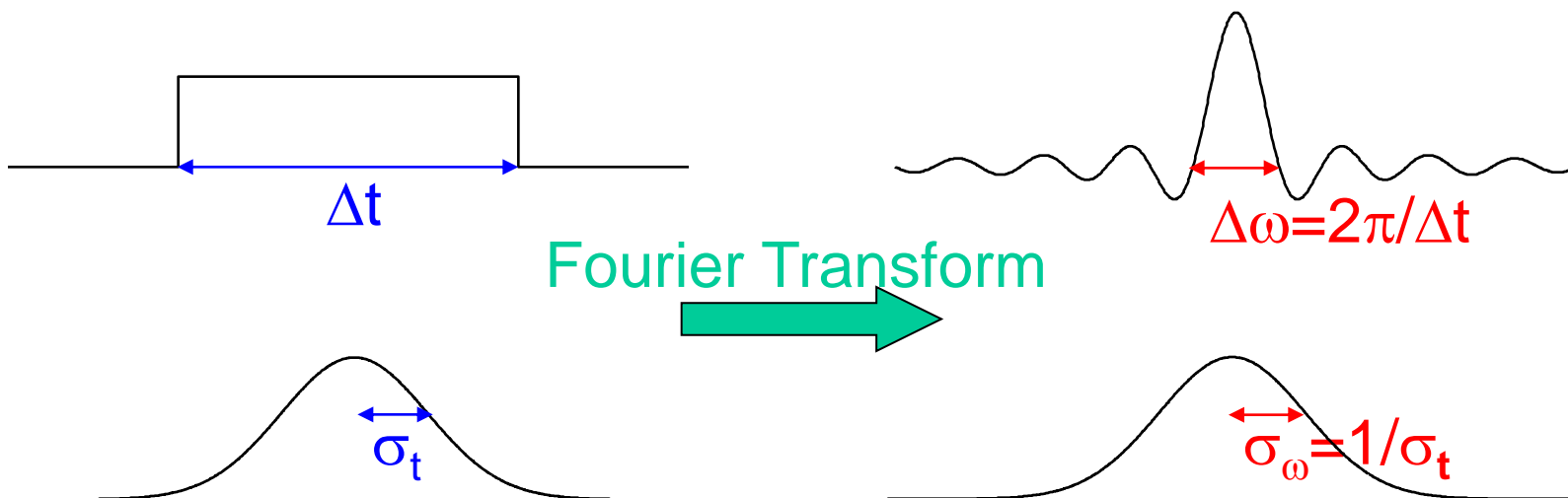
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- The photon distribution in the 6D phase space  $(x, y, x', y', t, \omega)$  gives us the full information on the properties of SR.
- Due to wave nature of light, however, we have two uncertainty relations to take care, which are well characterized by the Fourier transform.
- These relations imposes two restrictions on SR, **Fourier and Diffraction limits**.

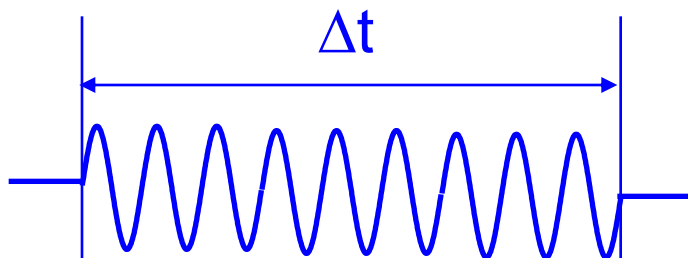
# Fourier Transform: Example

## Important Fourier Transform in SR Formulae

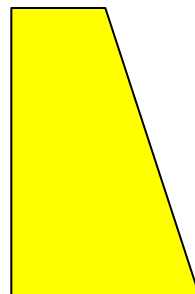
$F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t)e^{i\omega t}$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\frac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \text{sinc}(\omega \Delta t/2)$
$\frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\frac{\omega^2 \sigma_t^2}{2}\right)$



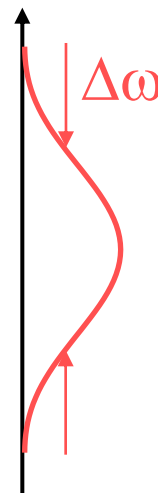
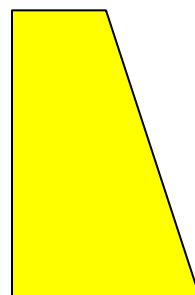
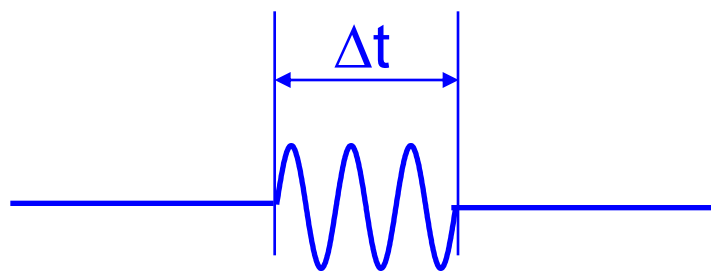
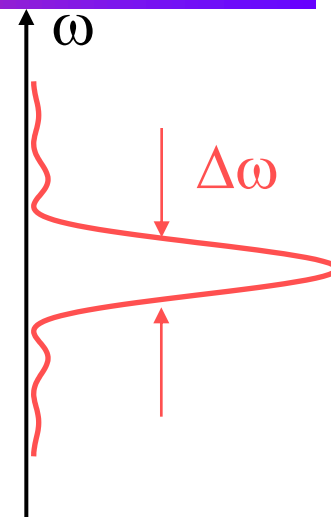
# Temporal Fourier Transform



Light with a finite time duration of  $\Delta t$



spectrometer = temporal Fourier transform



If the light is monochromatic, then  $\Delta\omega\Delta t = \text{const.}$

# Fourier Limit of Light

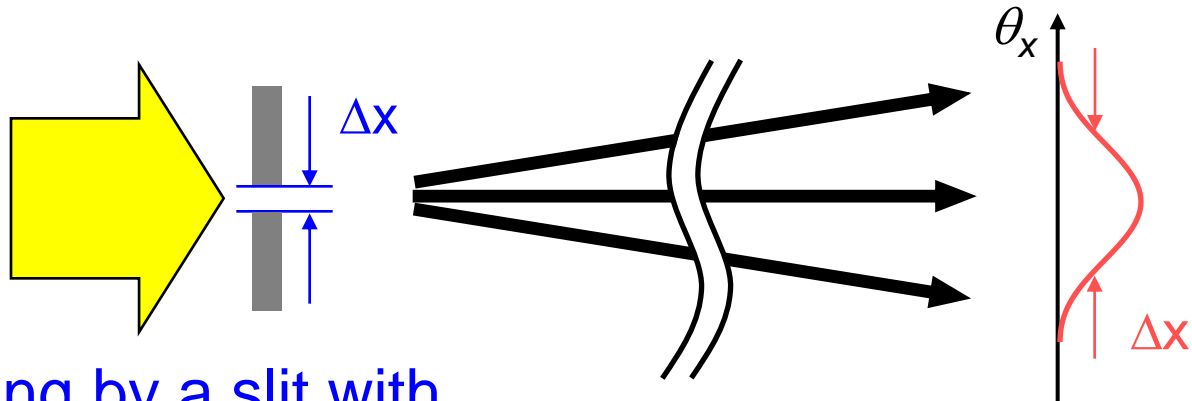
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- Temporal Fourier transform imposes

$$\Delta\omega \Delta t \geq \text{const.}$$

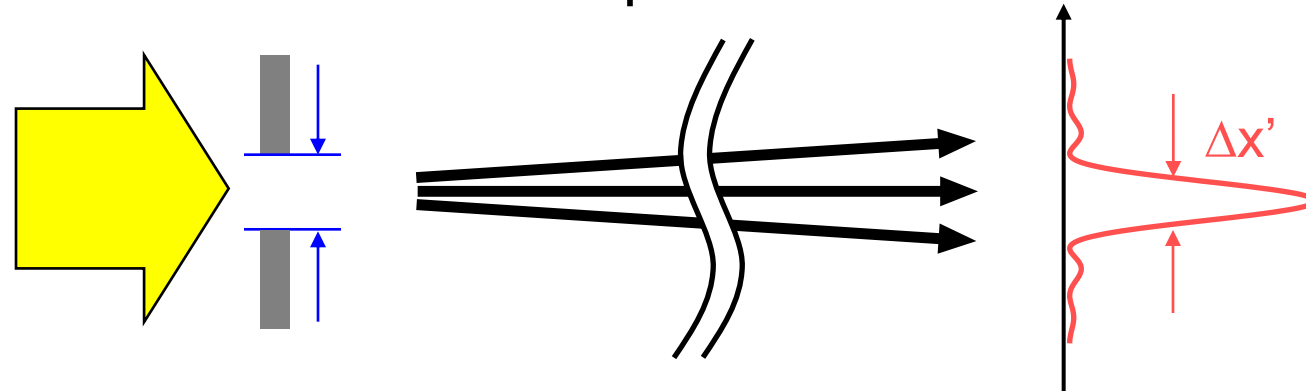
- Uncertainty of light in the  $(\omega, t)$  plane.
- When equality holds, light is said to be
  - Fourier-limited
  - Temporally coherent
- Important to understand the spectral properties of SR.

# Spatial Fourier Transform



Clipping by a slit with a finite width of  $\Delta x$

diffraction pattern in the far region = spatial Fourier transform



If the injected light is a plane wave, then  $\Delta x \Delta x' = \text{const.}$

# Diffraction Limit of Light

- Spatial Fourier transform imposes

$$\Delta x \Delta k_x \geq \text{const.}$$



$$\Delta x \Delta x' \geq \lambda \times \text{const.}$$

- Uncertainty of light in (x,x') plane
- When equality holds, light is said to be
  - Diffraction limited
  - Spatially coherent
- In the case of Gaussian beam,

$$\sigma_x \sigma_{x'} \geq \boxed{\lambda / (4\pi)} \text{ Natural emittance of light}$$

# Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

# SR: Light from a Moving Electron

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- Unlike the ordinary light source (sun, light bulb,... ), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different due to relativistic effects.
- What we have to take care is:
  1. Speed-of-light limit
  2. Squeezing of light pulse
  3. Conversion of the emission angles

# Speed-of-Light Limit

Within the framework of relativity, the velocity of an electron never exceeds the speed of light.

$$v/c = \beta = \sqrt{1 - \gamma^{-2}}$$

$$\sim 1 - \frac{1}{2\gamma^2}$$

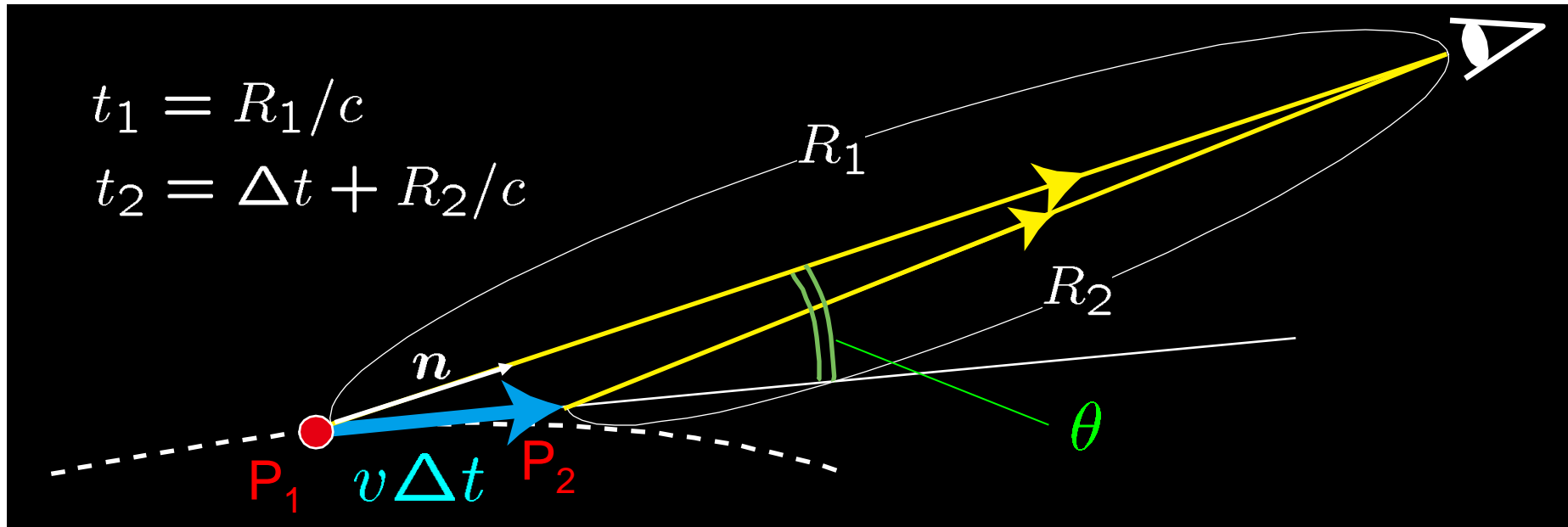
$$\gamma = \frac{E}{mc^2}$$

Energy	$\beta$
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.9999999998

:Lorentz Factor

(relative electron energy,  $mc^2=0.511\text{MeV}$ )

# Squeezing of Light Pulse Duration



$$R_2 = \sqrt{(R_1)^2 + (v\Delta t)^2 - 2R_1v\Delta t \cos\theta}$$

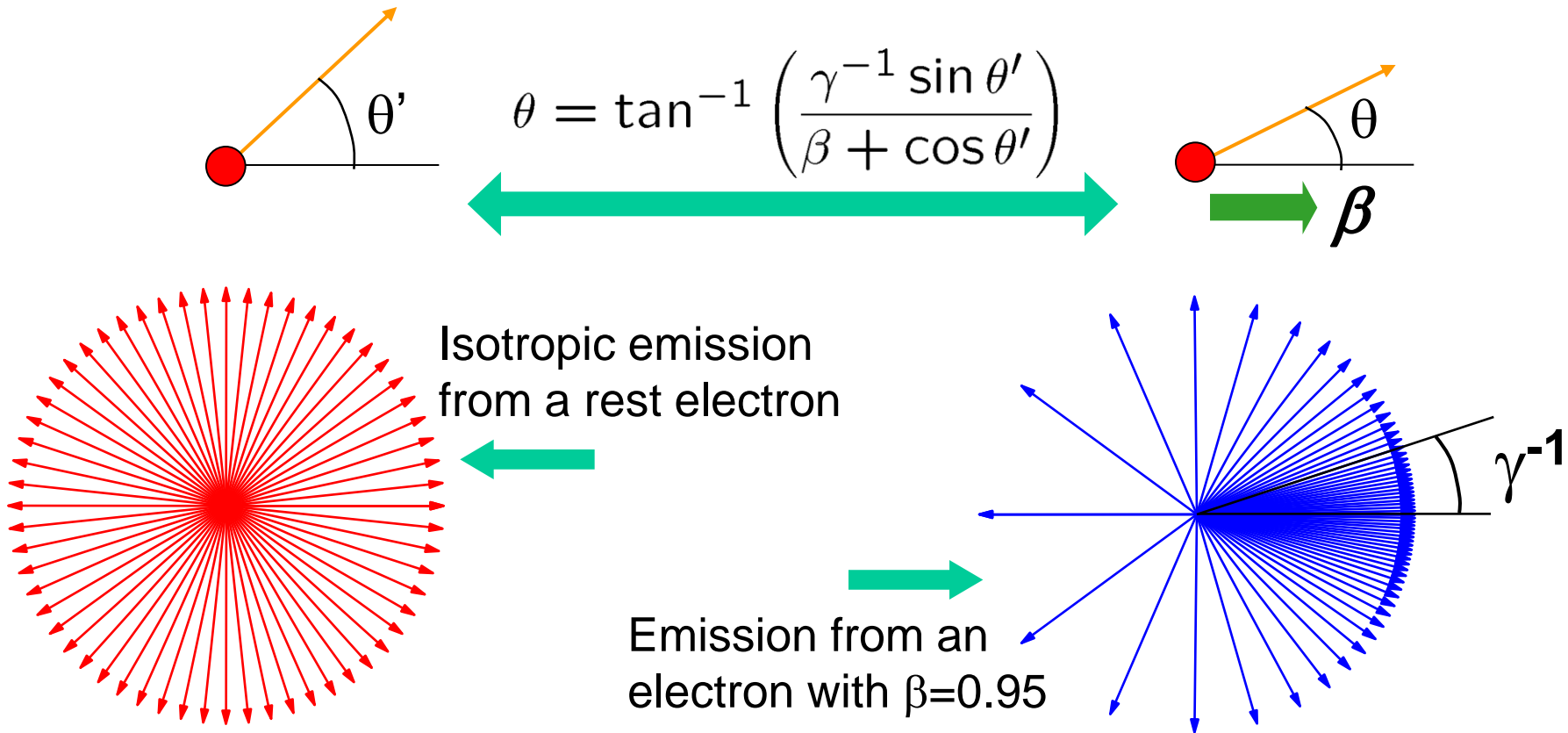
$$\sim R_1 - (v \cdot n)\Delta t$$

$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$

$$= \Delta t \boxed{(1 - \beta \cdot n)} = \boxed{\frac{\Delta t}{2\gamma^2}} \quad \gamma \gg 1, \theta = 0$$

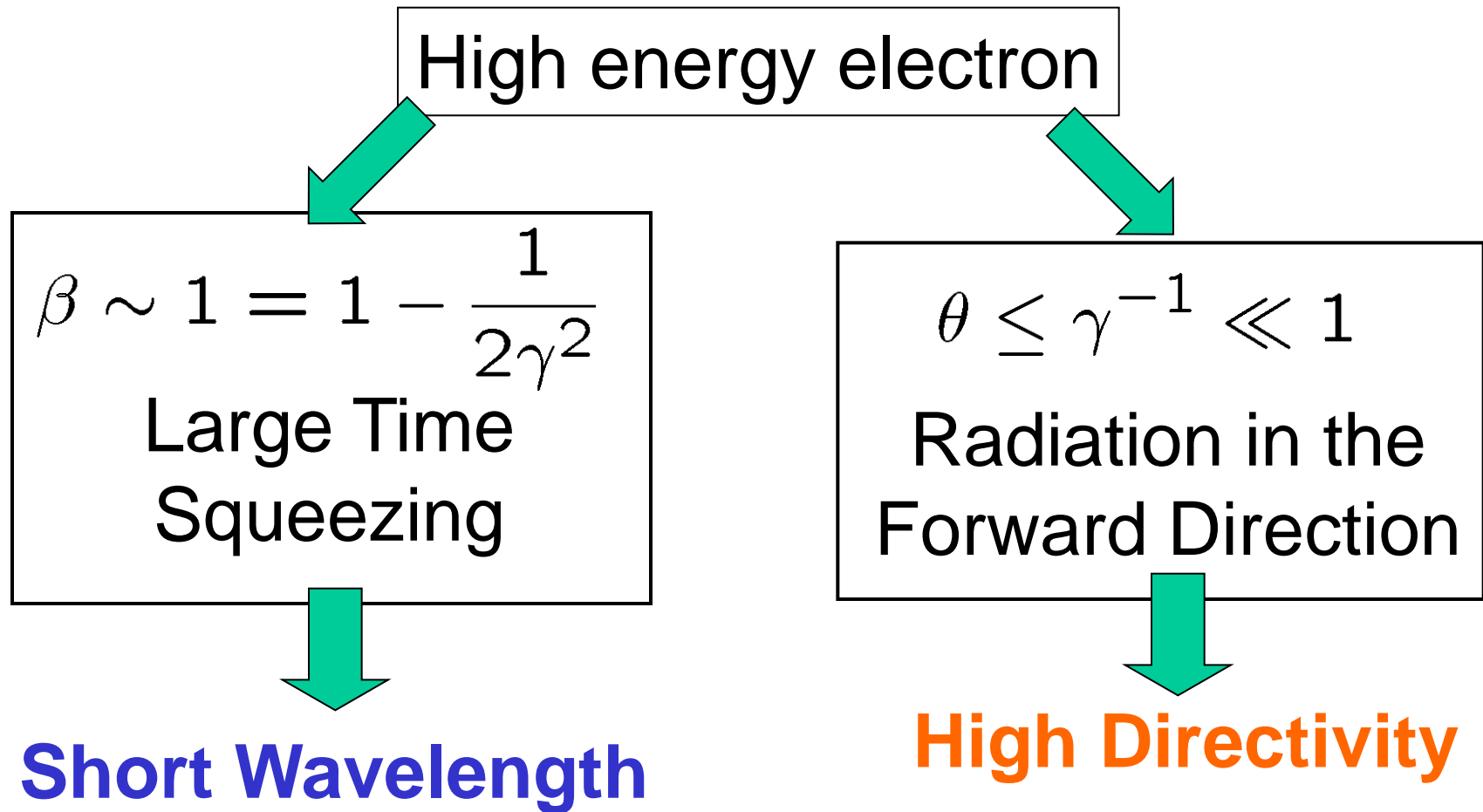
time squeezing

# Conversion of Emission Angles



Light emitted from a moving object ( $\beta \sim 1$ ) concentrates within  $\gamma^{-1}$

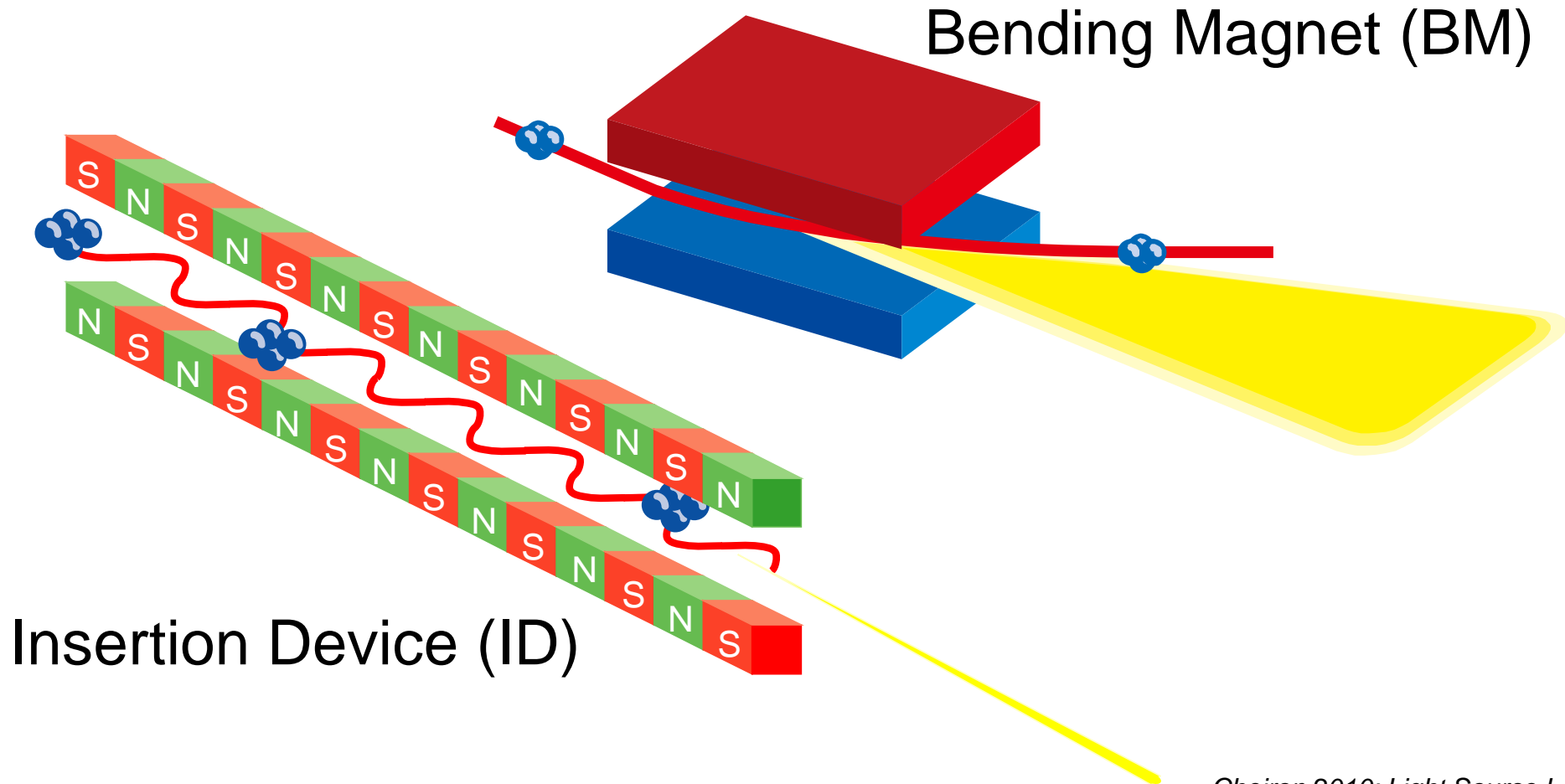
# SR from a High-Energy Electron



# Overview of SR Light Source

# What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



# Bending Magnet

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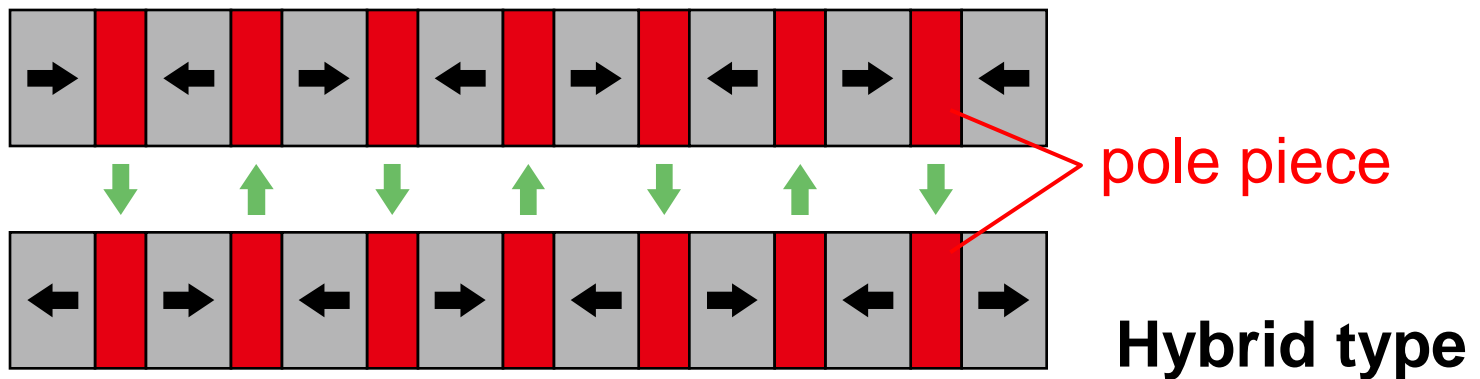
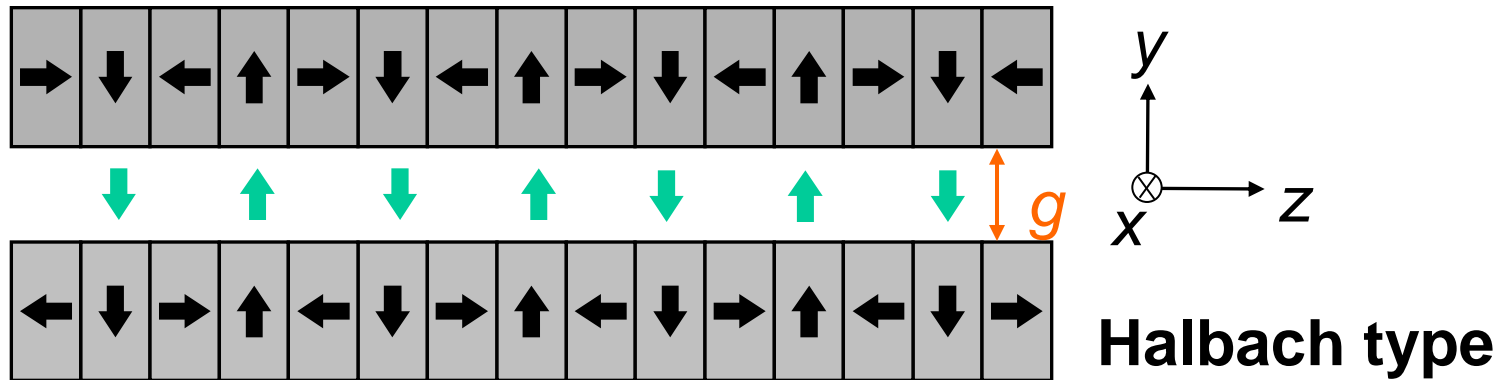
- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- EMs combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

# Insertion Device

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- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Classified into **wigglers** and **undulators**.

# Magnetic Circuit of IDs

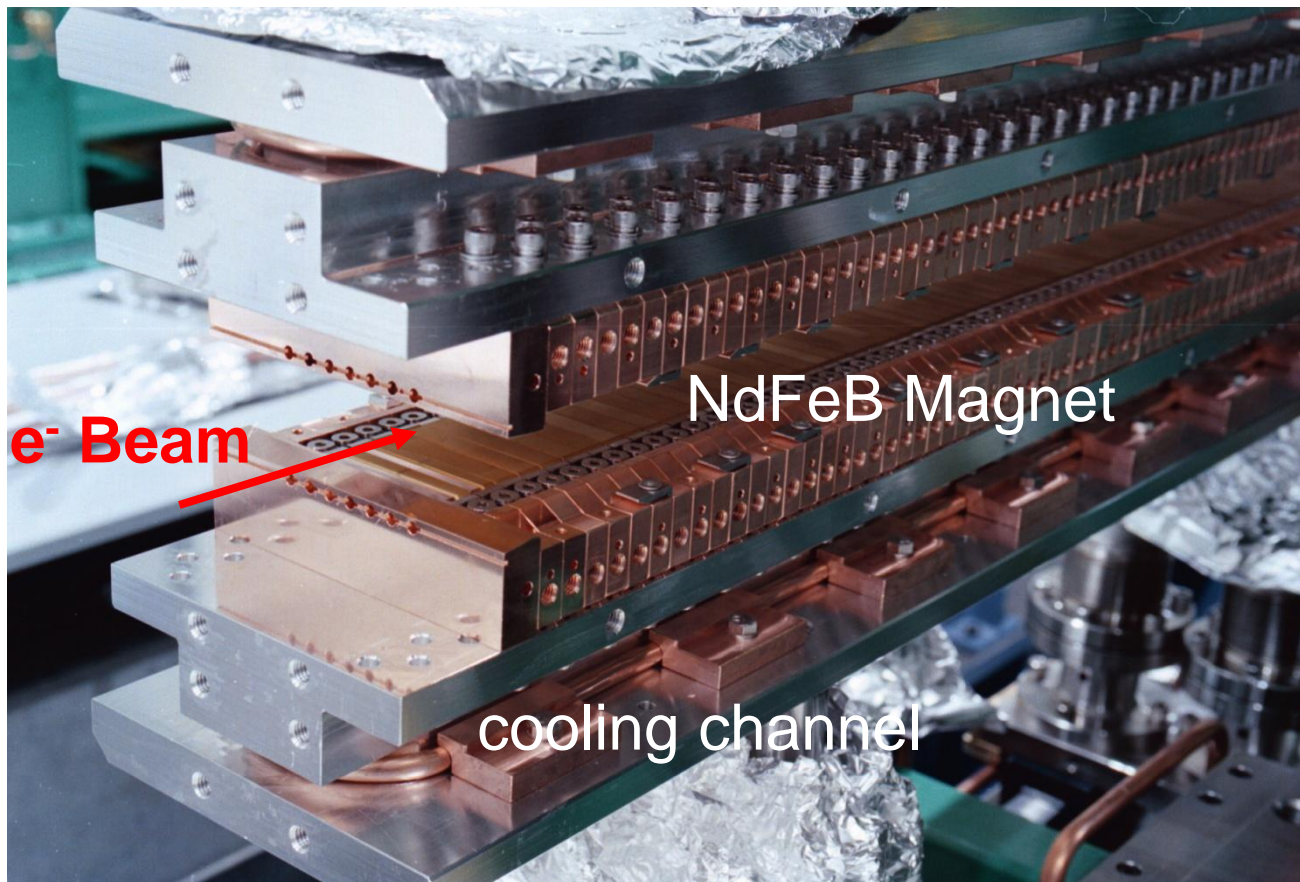


In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u) \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

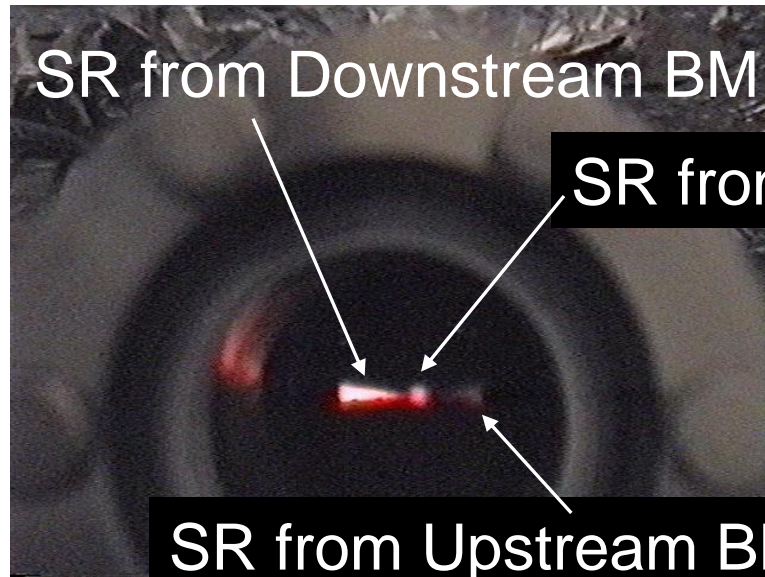
# Example of ID Magnets

## Halbach-type Magnet Array for SPring-8 Standard Undulators

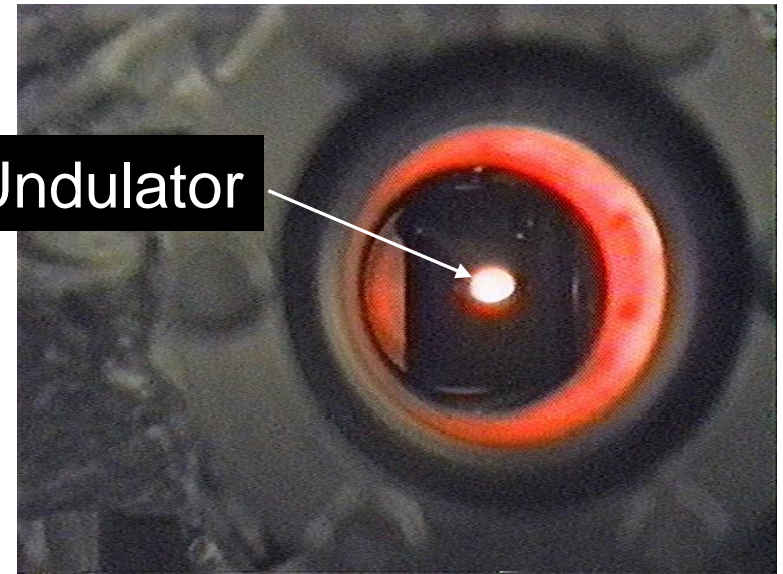


# Example of SR Image

BL41XU@SP-8, First Image of SR  
at Fluorescent Screen (<0.1mA)

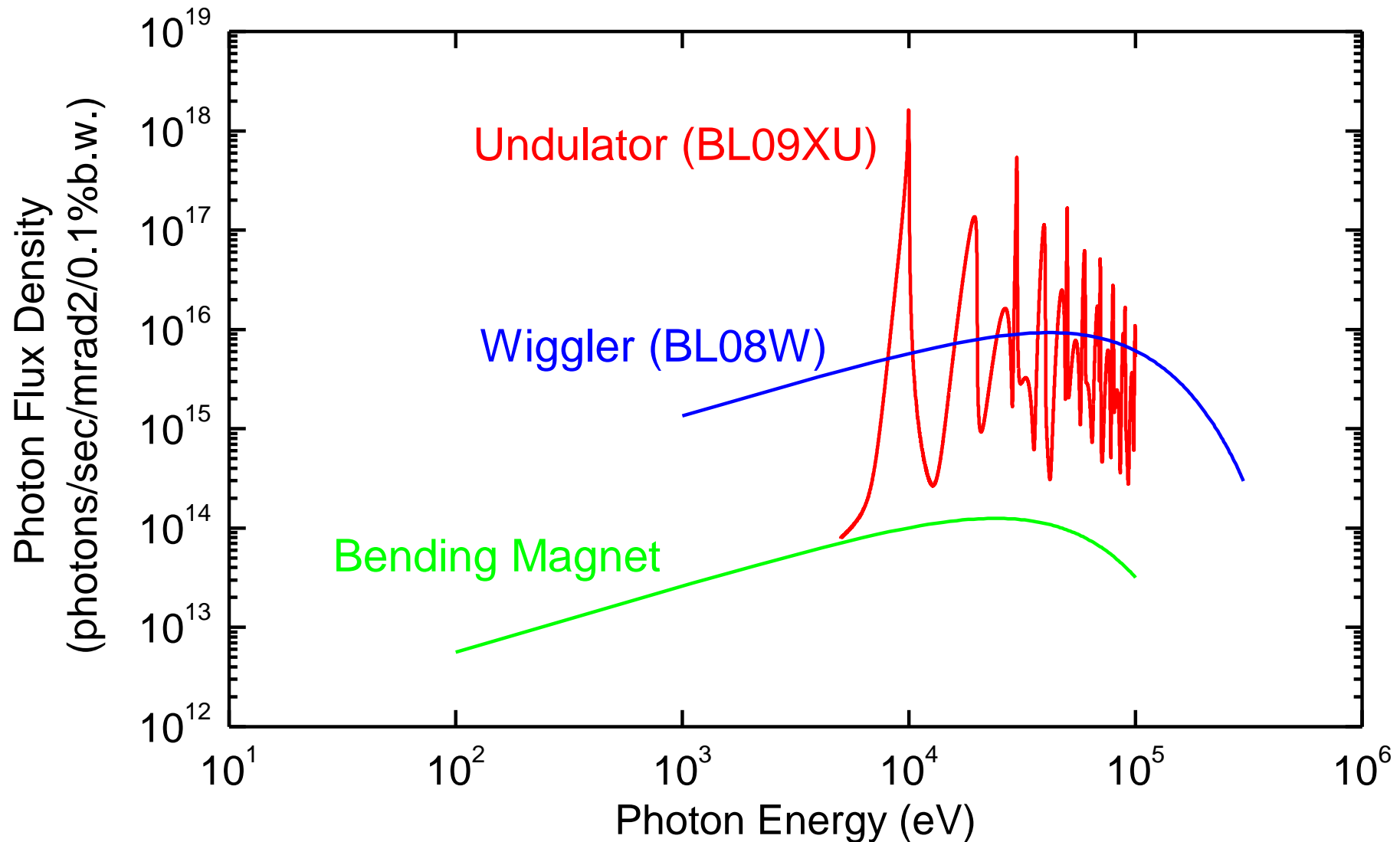


Undulator Gap = 50 mm



Undulator Gap = 20 mm

# Comparison of Light Sources

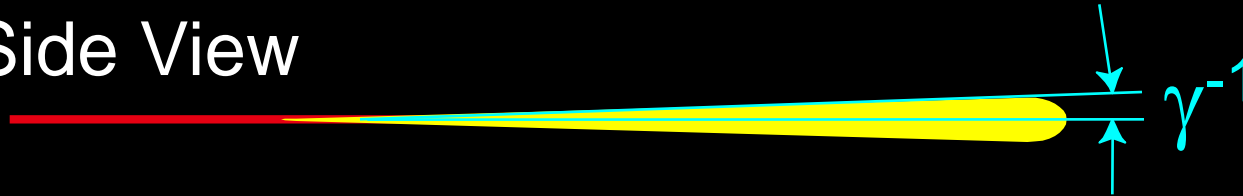


# Characteristics of SR (1)

- Radiation from BMs

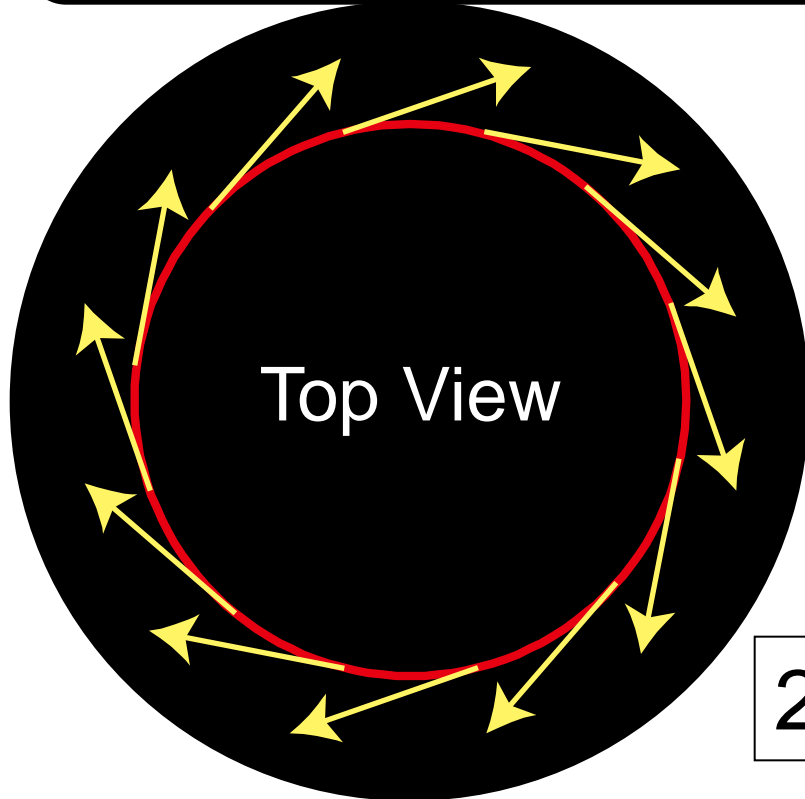
# Directivity of BM Radiation

Side View



High directivity in the vertical plane  
 ( $\sigma_y \sim \gamma^{-1} \sim 64 \mu\text{rad} @ \text{SP-8}$ )

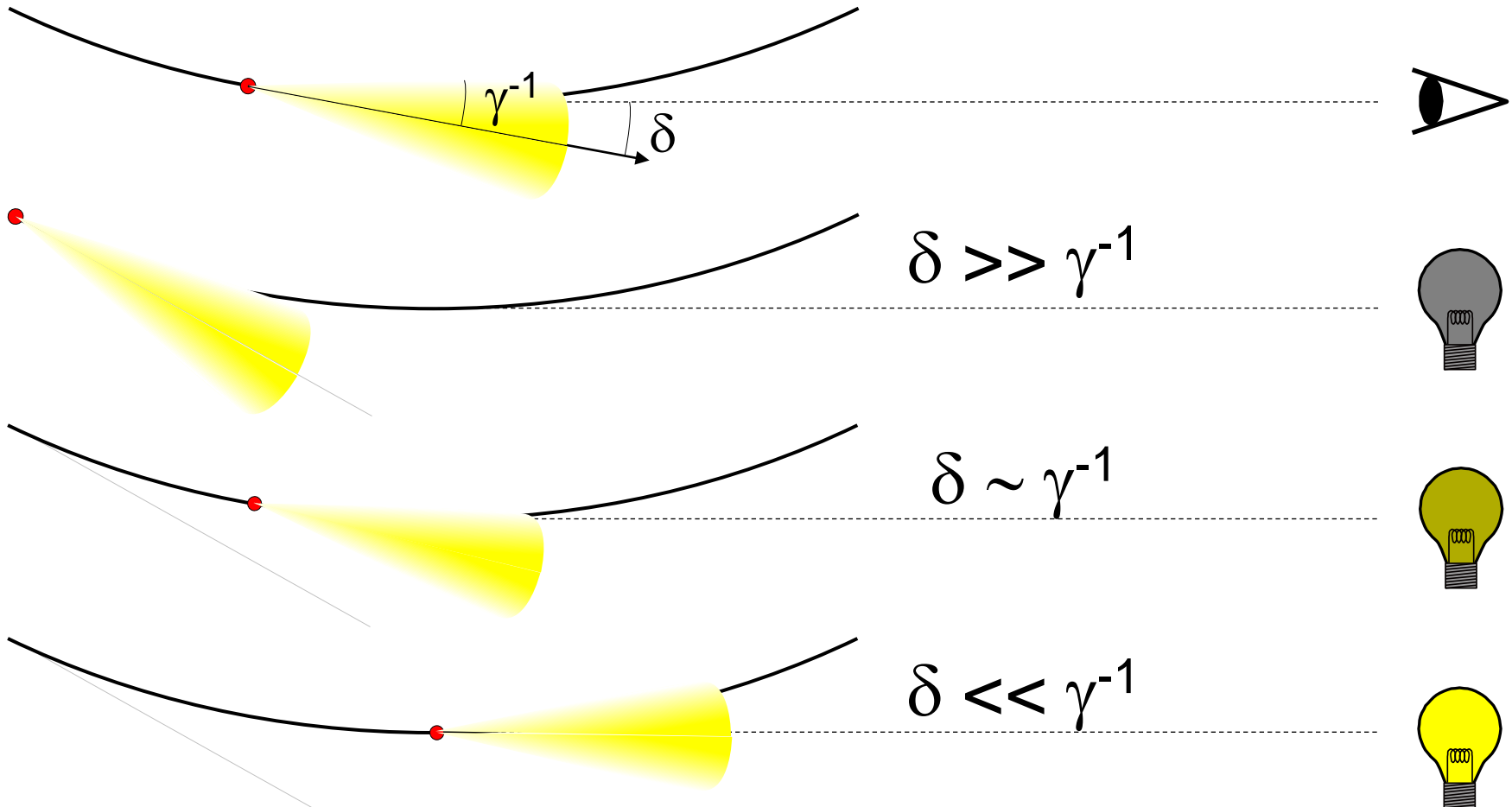
Top View



Isotropic in the orbital plane

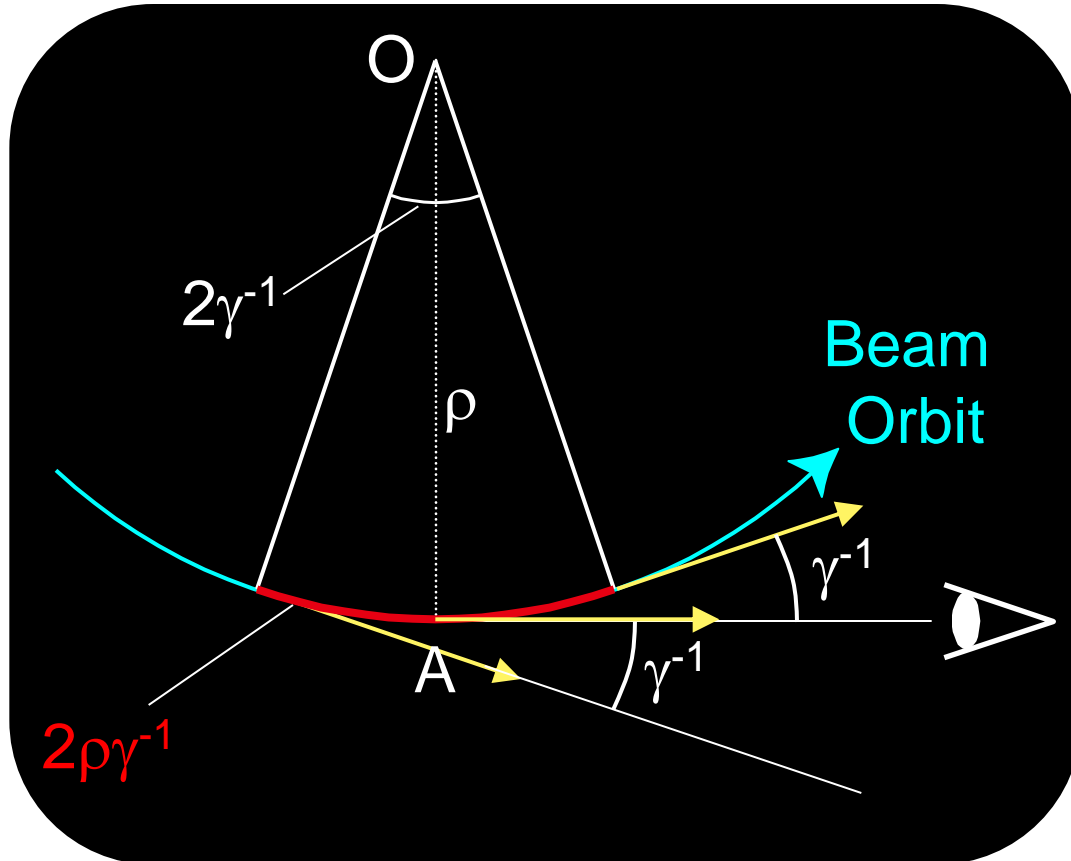
2-dimensional directivity

# Spectrum of BM Radiation (1)



Photons emitted when  $\delta < \gamma^{-1}$   
are detected by the observer

# Spectrum of BM Radiation (2)



Major contribution of radiation is from the portion painted red



Pulse duration for  $e^-$   
 $\Delta t = 2\rho\gamma^{-1}/c$

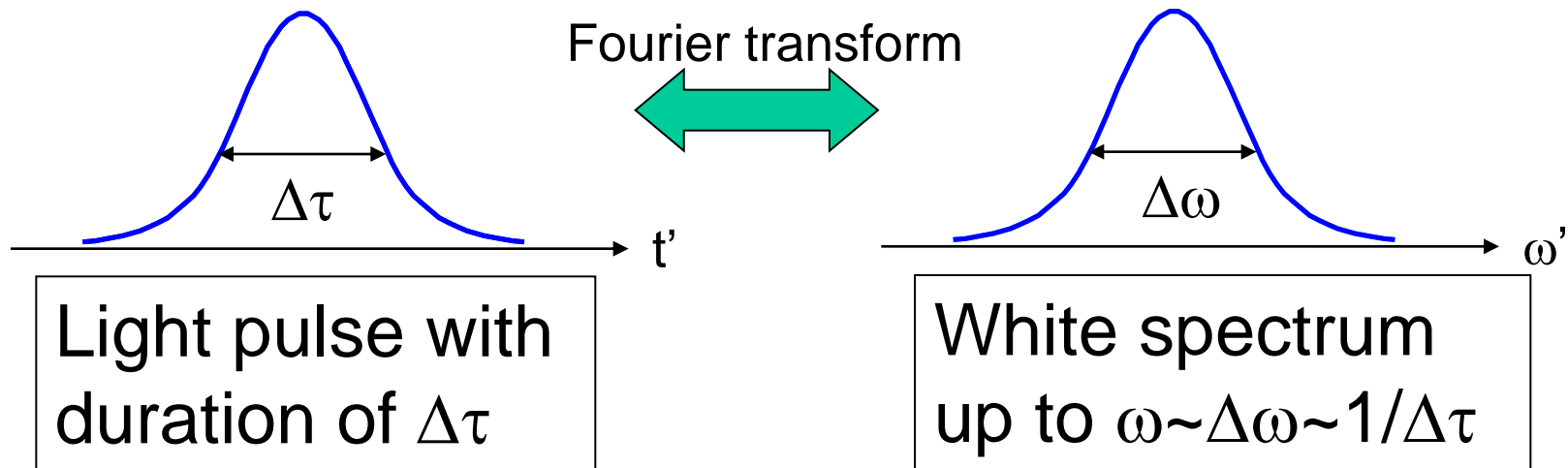


squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

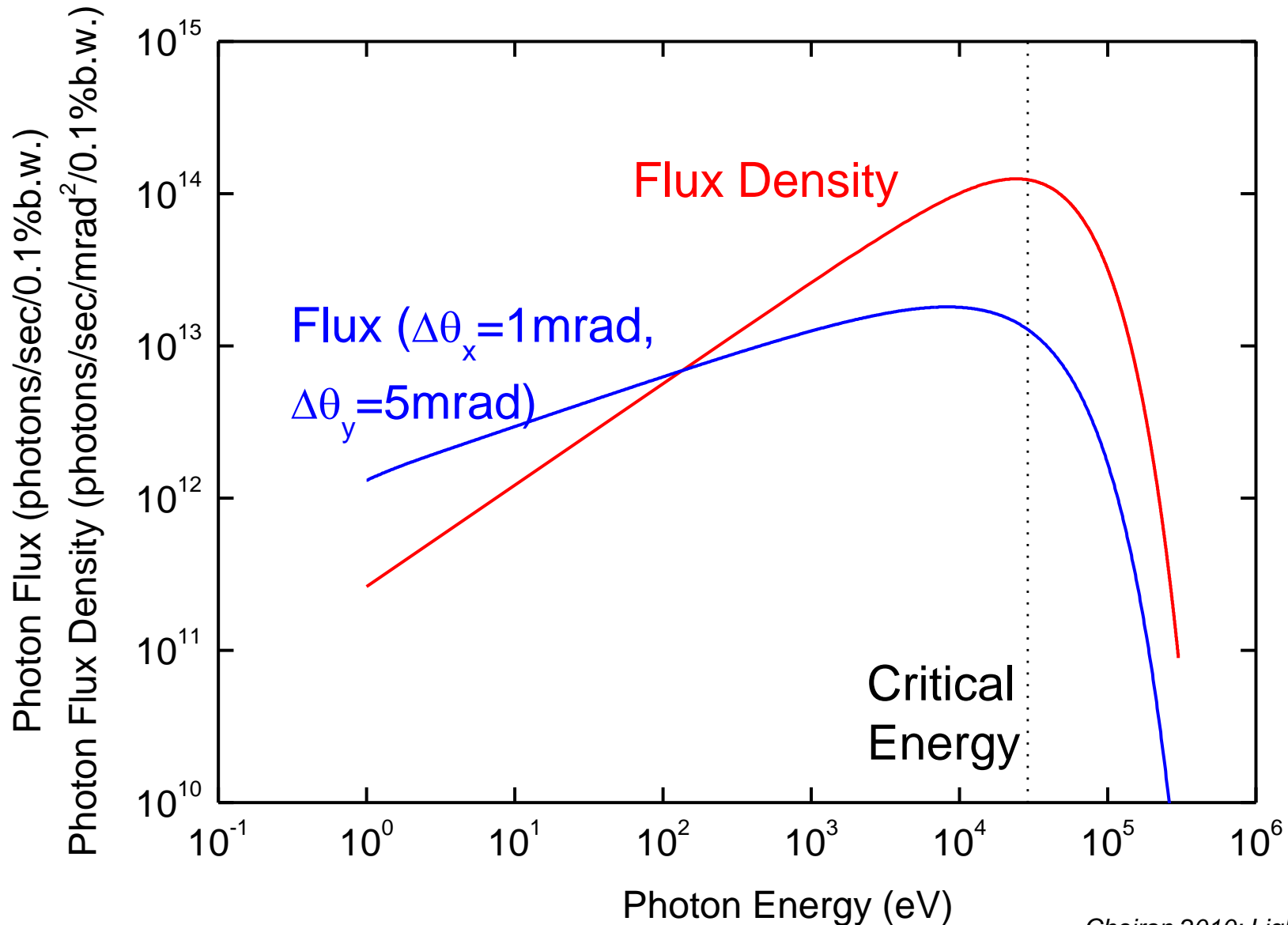
# Spectrum of BM Radiation (3)



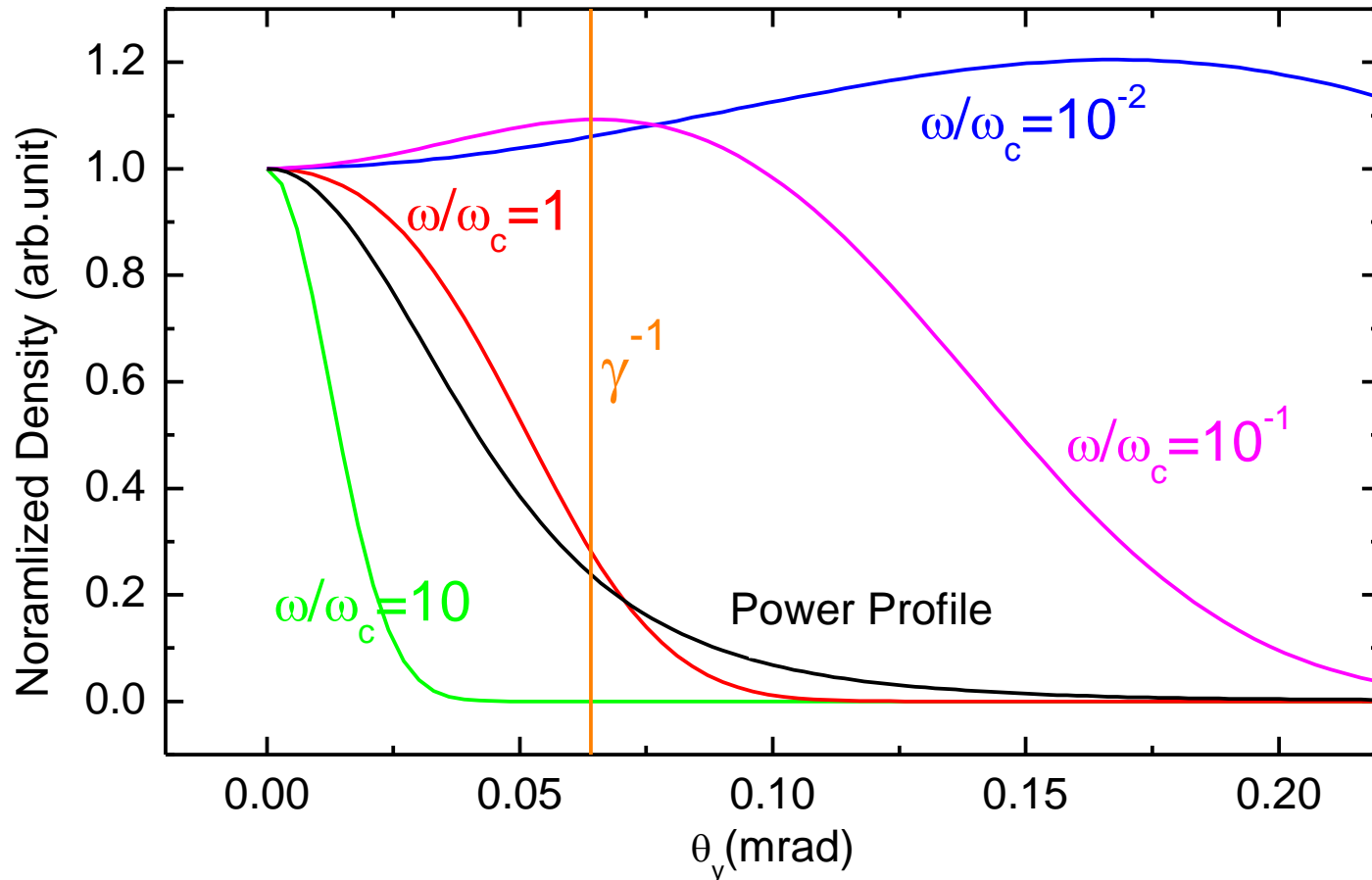
- By definition,  $\omega_c = (3/2)/\Delta\tau = 3\gamma^3 c/2\rho$  is called “critical frequency” of SR, which gives a criterion of the maximum energy of SR from a BM.
- In practical units,

$$\hbar\omega_c(\text{keV}) = 0.665 E_e^2(\text{GeV}) B(\text{T})$$

# Example of Spectrum



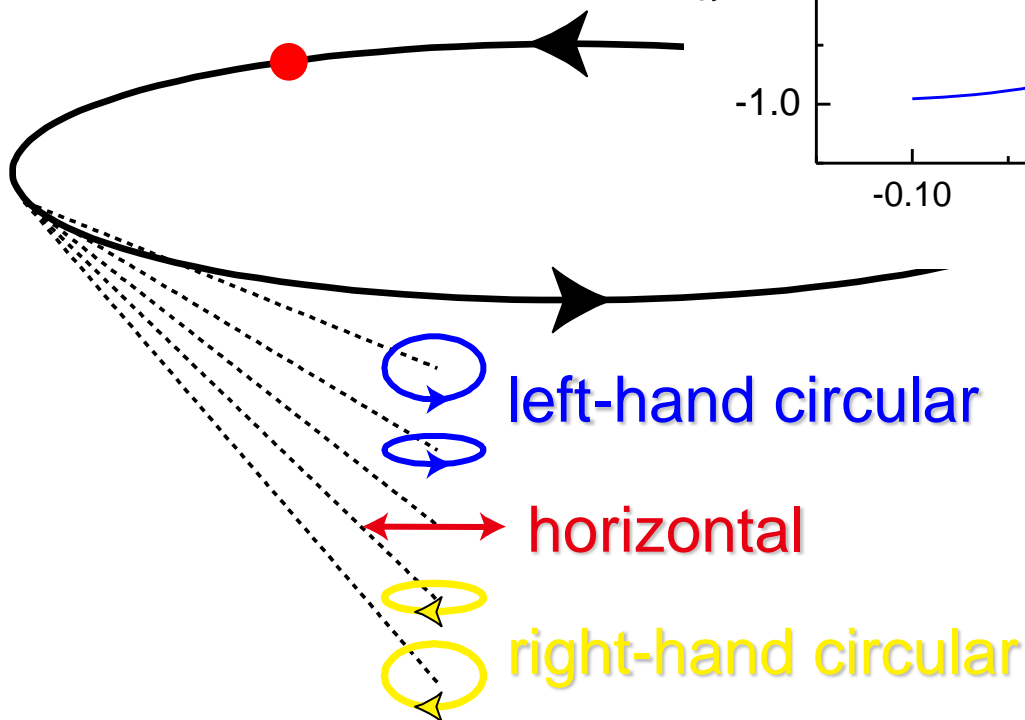
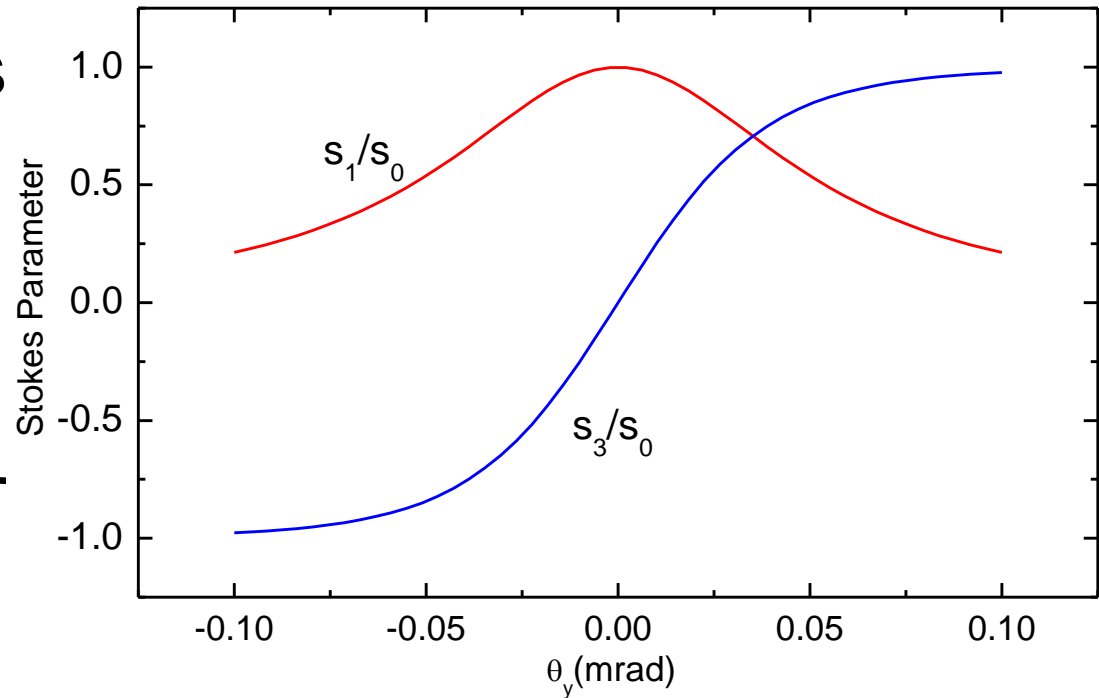
# Angular Profile of BM Radiation



- power profile  $\sim$  flux profile @  $\omega/\omega_c = 1$
- larger angular divergence for lower energy

# Polarization of BM Radiation

Stokes parameters  
of BM radiation  
along vertical axis



Polarization state  
reflects the apparent  
motion of electron.