Light Source I

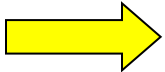*Takashi TANAKA (RIKEN SPring-8 Center)*

CONTENTS

Light Source I
- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)

Light Source II
- Characteristics of SR (2)
- Practical Knowledge on SR
- Numerical Characterization of SR

Introduction

SR Facility and Light Source

- ## SR: Definition
  - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- ## SR Facility
  - Accelerators to generate a high-energy electron beam
  - **Magnetic devices (SR light source) to generate intense SR**
  - Optical elements (monochromators, mirrors,..)
  - Experimental stations

SR as a Probe for Research

- SR has a lot of advantages over other conventional light sources
  - Highly collimated (laser-like)
  - Wavelength tunability
  - Polarization
  - .....
- However, the total radiation power does not differ significantly.

Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
  - Why we need SR?
  - Physical quantity of light
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet configuration
- Characteristics of SR (1)
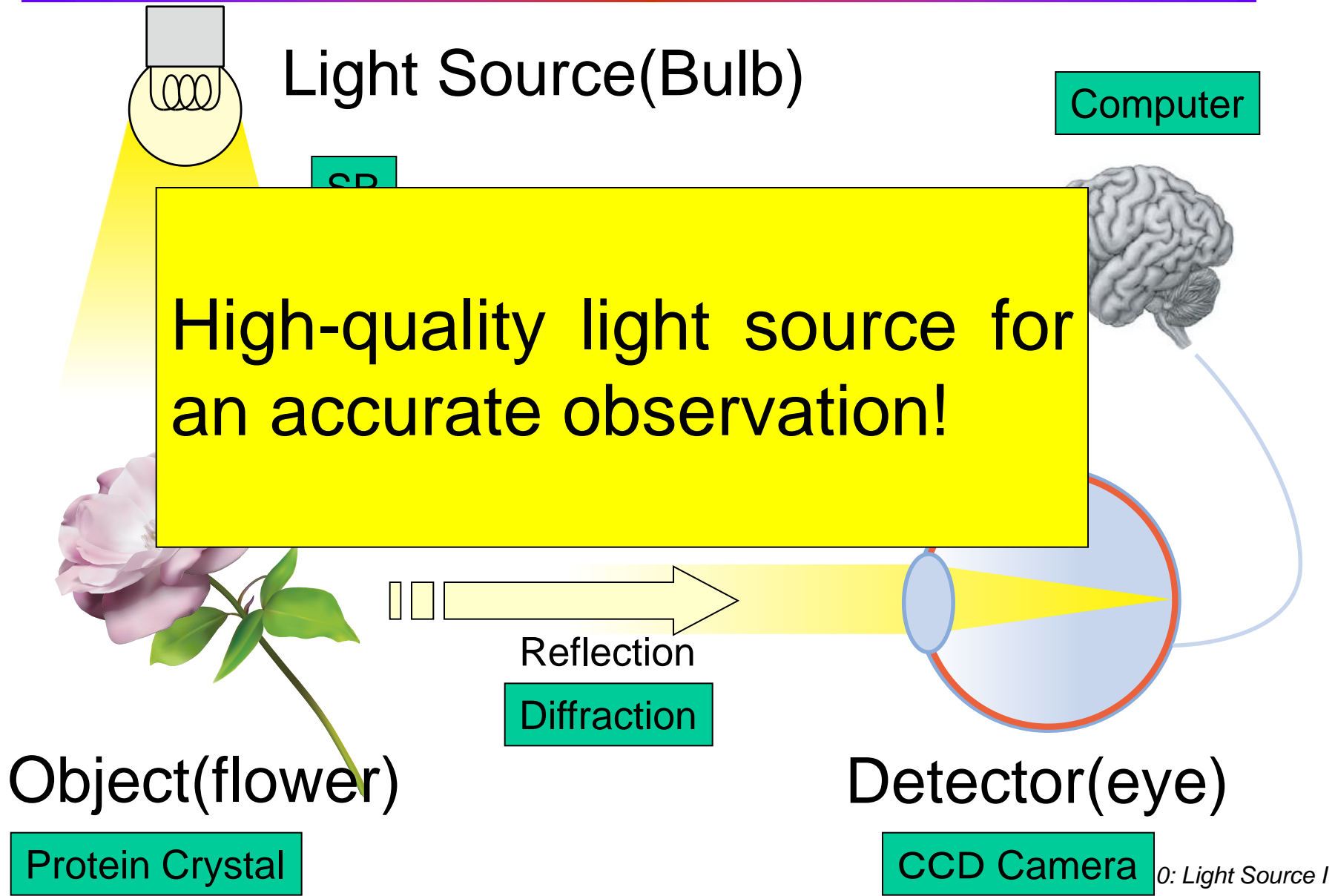 - Radiation from bending magnets

Topics in This Lecture (2)

- ## Characteristics of SR (2)
  - Electron Trajectory in IDs
  - Radiation from wigglers
  - Radiation from undulators

- ## Practical Knowledge on SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Definition of undulators and wigglers

- ## Numerical Characterization of SR
  - Introduction to "SPECTRA" and examples

Fundamentals of Light and SR

- Why we need SR?

- Physical Quantity of Light
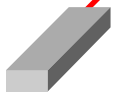
- Uncertainty of Light

- SR: Light from a Moving Electron

Observation with Light

Light Source(Bulb)

Computer

SR

High-quality light source for an accurate observation!

Reflection

Diffraction

Object(flower)

Detector(eye)

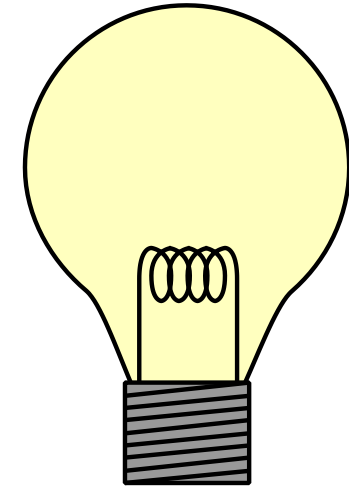Protein Crystal

CCD Camera

Which Quality is Better?

Specs of SPring-8
- E = 8GeV
- I = 100mA
- L = 1500m

1mW Laser (pointer)   100W Bulb

Lighting equipment in a room: Bulb
Pointer during a presentation: Laser

Depends on the Object!

How to Define the Quality of Light?(1)

- The performance of the light source depends on the dimension of the object and the method to detect light.

- For observation, the photons emitted by the light source should be

  – <u>illuminated on the object for interaction</u>

  – <u>recognized by the detector for analysis</u>

Quality of Light Source:
How efficiently the above conditions are satisfied?

How to Define the Quality of Light?(2)

Important Features of the Light Source

	Object		Related Items
	Flower	Protein	
Radiation Power	◎	◯	# Emitted Photons
Source Size	✕	◎	Illuminated Area
Directivity	△	◎	
Monochromaticity	△	◎	Accuracy of Analysis

Brilliance

What is Brilliance?

Brilliance(photons/sec/mm^2/mrad^2/0.1%B.W.)

~ $\dfrac{\text{Total Power}}{\text{Source Size x Angular Divergence x Band Width}}$

- Brilliance specifies the quality of light for observation of microscopic objects.

- The brilliance of a light source with a high total power is not necessarily high.

Examples of Brilliance

|	Bulb	Laser Pointer
Total Power (W)	100	10^{-3}
Angular Div. (mrad^2)	$4\pi \times 10^6$	1
Source Size: (mm^2)	10^2	1
Bandwidth: (%)	100	0.01
Brilliance (photons/sec/....)	$\sim 10^8$	$\sim 10^{16}$

Laser is the best light source to observe the microscopic object!

X ray as a Probe

- Definition (not unique)
  - Electromagnetic wave (= light) with λ of 10 nm(10^{-8} m) ~ 0.1 Å (10^{-11} m)

- Properties
  - High Energy/Photon
  - High Penetration (Roentgen etc..)

- Application to Microscopic Objects
  - X-ray Diffraction
  - Fluorescent X-ray Analysis
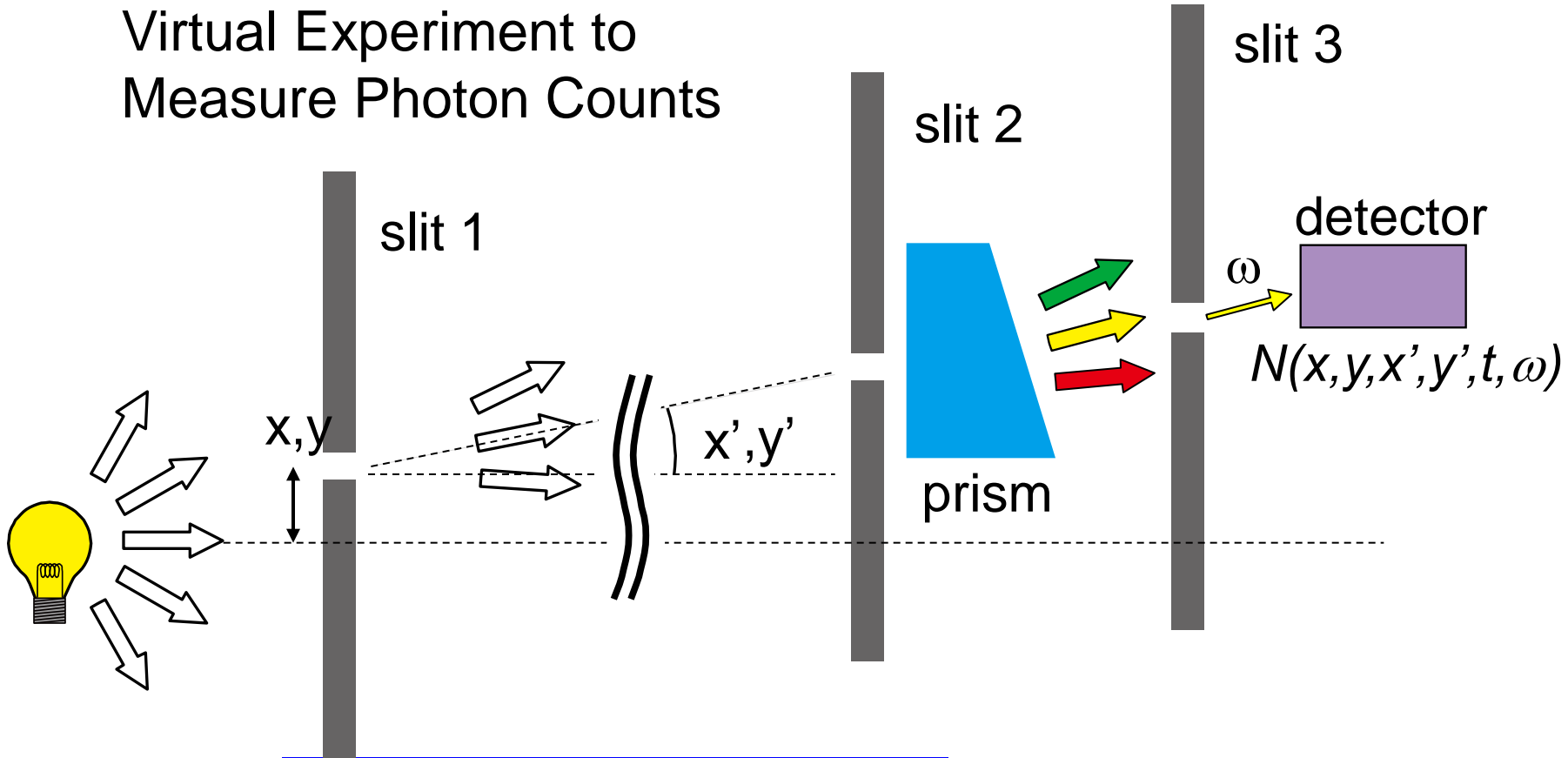
- No Practical Lasers!!
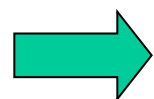
➡ Synchrotron Radation(SR)

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Phase Space

Virtual Experiment to
Measure Photon Counts

slit 1

slit 2

slit 3

detector

x,y

x',y'

prism

ω

$N(x,y,x',y',t,\omega)$

$$n = \frac{N(x,y,x',y',\omega,t)}{\Delta x \Delta y \Delta x' \Delta y' \Delta t \Delta \omega / \omega} \equiv \frac{N}{\Delta \Omega}$$

Av. Photon Density

Volume in 6-D
Phase Space

Brilliance (Brightness)

- Brilliance (photons/sec/mm^2/mrad^2/0.1%B.W.) is defined as the photon density in the 6D phase space, i.e.,

$$B = \lim_{\Delta\Omega\to0} n = \frac{d^6 N(x, y, x', y', t, \omega)}{dx\,dy\,dx'\,dy'\,dt\,d\omega/\omega}$$

- In practice, $\Delta\Omega$ can never be 0 due to uncertainty of light, thus brilliance is not a physical quantity that can be actually measured.

Photon Flux and Flux Density

- Removing the 1ˢᵗ slit gives the angular flux density (photons/sec/mrad²/0.1%B.W), i.e.,

$$\frac{d^2 F}{dx' dy'} = \iint B \, dx \, dy$$

- Removing the 1ˢᵗ & 2ⁿᵈ slits gives the total flux (photons/sec/0.1%B.W), i.e.,

$$F = \iiiint B \, dx \, dy \, dx' \, dy'$$

- Estimation of number of photons to be delivered to the sample.

Radiation Power and Power Density

- Removing the 1st & 3rd slits gives the angular power density (W/mrad²), i.e.,

$$\frac{d^2 P}{dx' dy'} = 10^3 Q_e \hbar \int \frac{d^2 F}{dx' dy'} d\omega$$
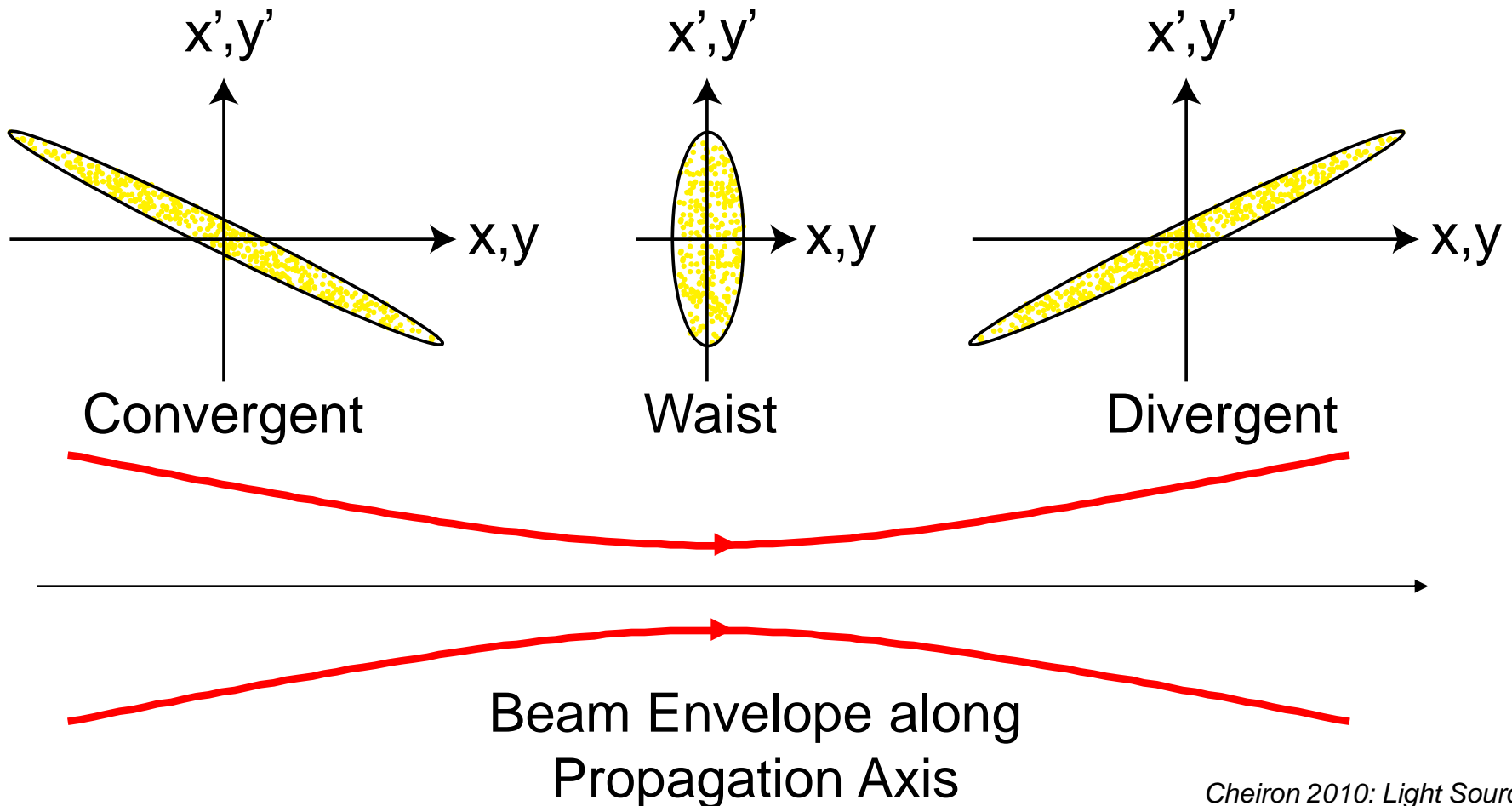
conversion from photons/sec/0.1%B.W. to W

- Removing all the slits gives the total power (W), i.e.,

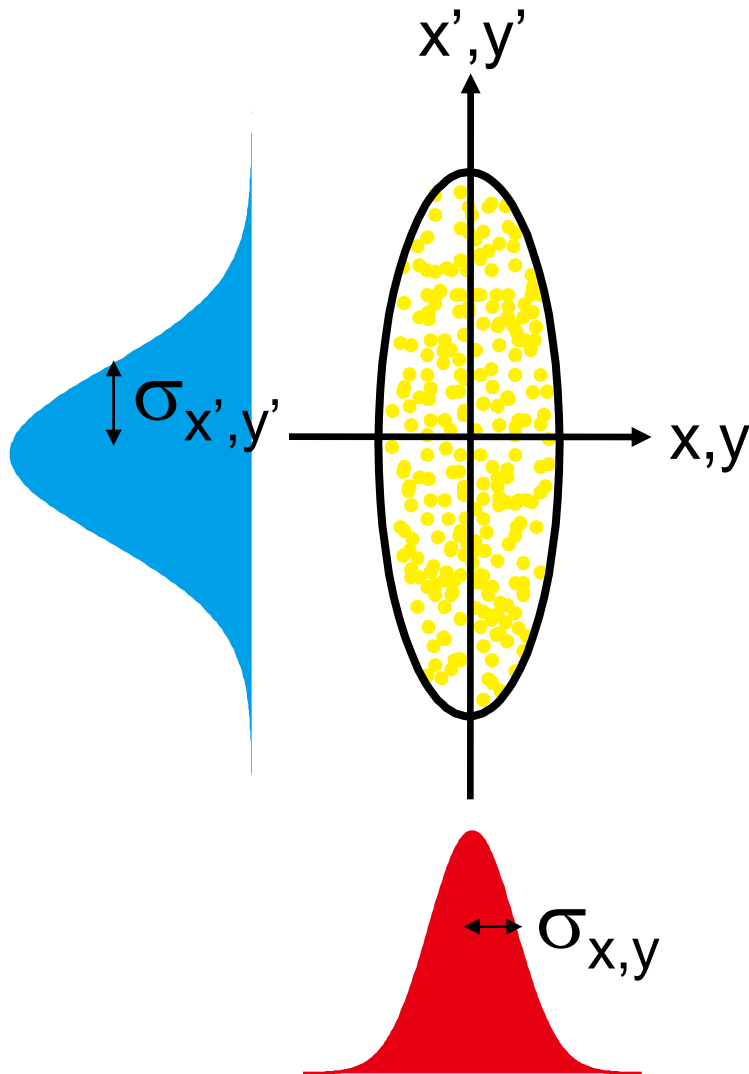$$P = 10^3 Q_e \hbar \iiint \frac{d^2 F}{dx' dy'} d\omega \, dx' \, dy'$$

- Estimation of heat load on BL components.

Photons in 4D Phase Space

- Photon distribution in the 4-D phase space at different longitudinal positions.

Convergent   Waist   Divergent

Beam Envelope along
Propagation Axis

Beam Size, Divergence, Emittance

- Beam size ($\sigma_{x,y}$) is defined as the beam envelope at the beam waist position.

- Angular divergence ($\sigma_{x',y'}$) is constant along the axis of propagation, as far as no optical elements are present.

- Emittance ($\varepsilon_x, \varepsilon_y$) is defined as $\sigma_{x,y} \times \sigma_{x',y'}$, which is equal to the area of the phase ellipse divided by π.

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron
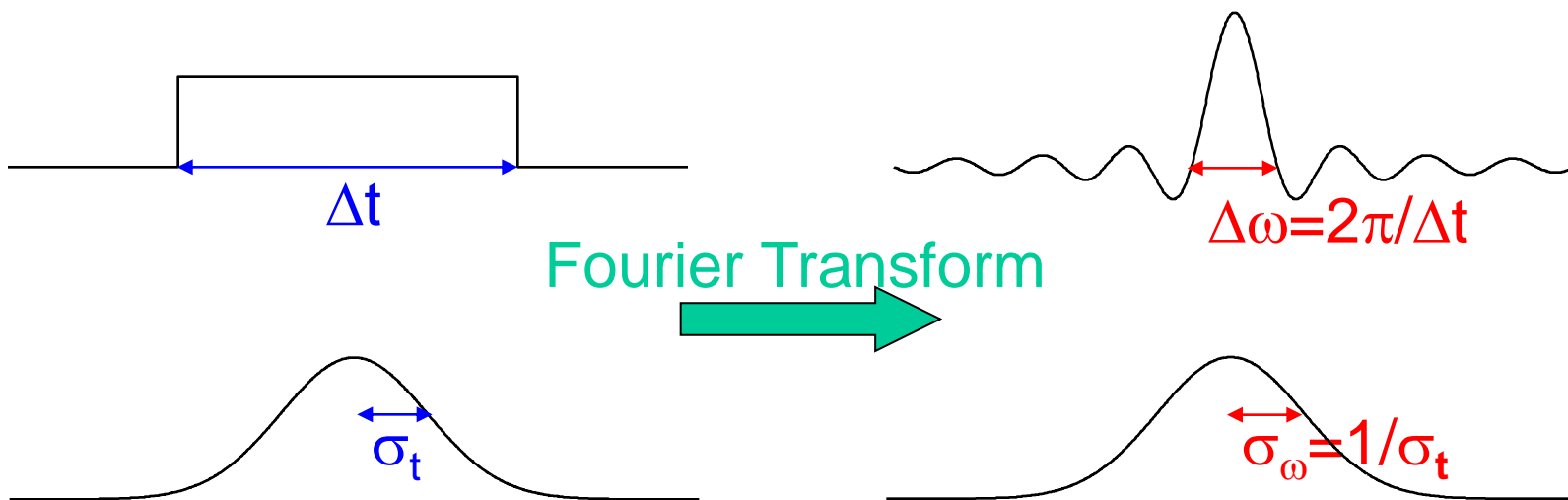
Uncertainty of Light

- The photon distribution in the 6D phase space (*x,y,x',y',t,ω*) gives us the full information on the properties of SR.

- Due to wave nature of light, however, we have two uncertainty relations to take care, which are well characterized by the Fourier transform.

- These relations imposes two restrictions on SR, Fourier and Diffraction limits.
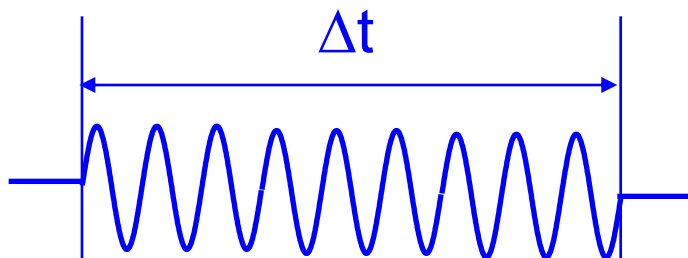
Fourier Transform: Example

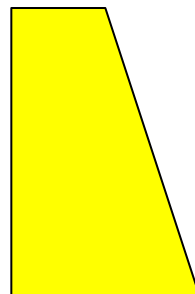## Important Fourier Transform in SR Formulae

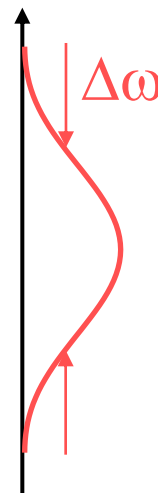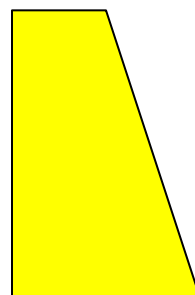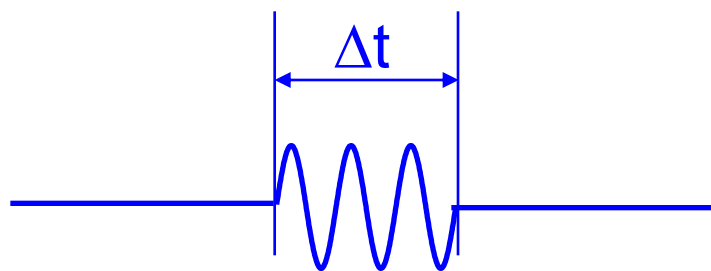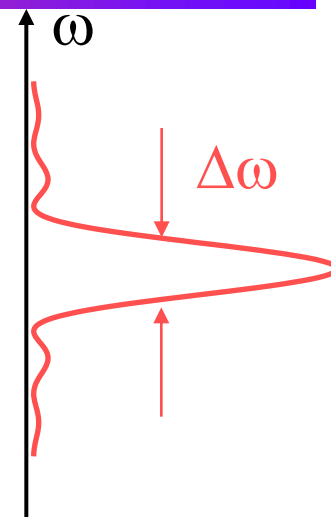| $F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t)\mathrm{e}^{i\omega t}$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\dfrac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \mathrm{sinc}(\omega \Delta t/2)$
$\dfrac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\dfrac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\dfrac{\omega^2 \sigma_t^2}{2}\right)$

Δt

Fourier Transform

$\Delta\omega = 2\pi/\Delta t$

σ_t

$\sigma_\omega = 1/\sigma_t$

Temporal Fourier Transform

Δt

Light with a finite
time duration of Δt

spectrometer =
temporal Fourier transform

ω

$\Delta\omega$

Δt

$\Delta\omega$

If the light is monochromatic,
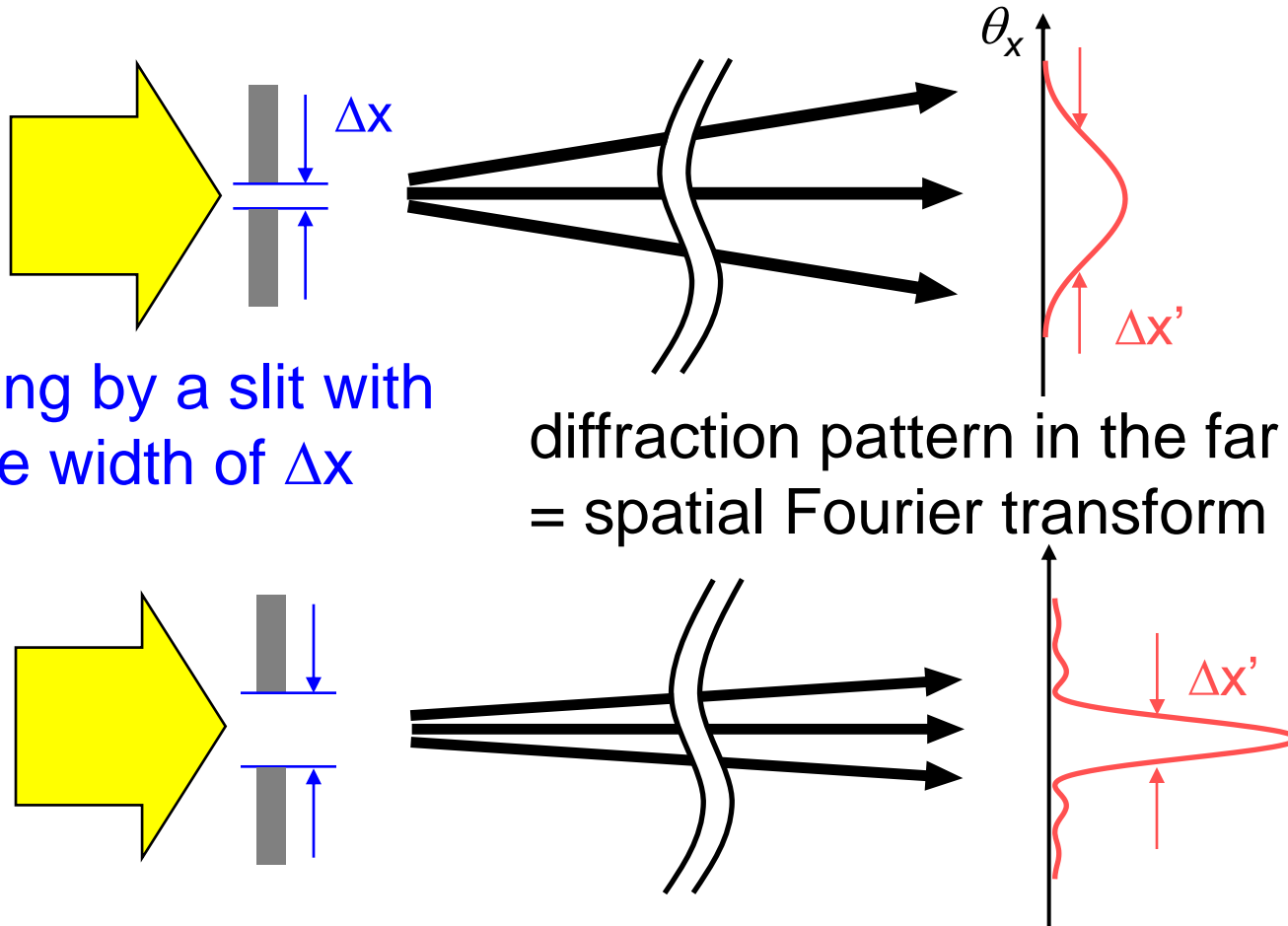then $\Delta\omega\Delta t$=const.

Fourier Limit of Light

- Temporal Fourier transform imposes

$$\Delta\omega\Delta t \geq \text{const.}$$

- Uncertainty of light in the (ω,t) plane.

- When equality holds, light is said to be
  - Fourier-limited
  - Temporally coherent

- Important to understand the spectral properties of SR.

Spatial Fourier Transform

θ_x

Δx

$\Delta x'$

Clipping by a slit with a finite width of Δx

diffraction pattern in the far region = spatial Fourier transform

$\Delta x'$

If the injected light is a plane wave, then $\Delta x \Delta x' =$ const.

Diffraction Limit of Light

- Spatial Fourier transform imposes

$$\Delta x \Delta k_x \geq \text{const.}$$

$$\boxed{k_x = (2\pi/\lambda)x'} \longrightarrow \quad \Delta x \Delta x' \geq \lambda \times \text{const.}$$

- Uncertainty of light in (x,x') plane
- When equality holds, light is said to be
  - Diffraction limited
  - Spatially coherent
- In the case of Gaussian beam,

$$\sigma_x \sigma_{x'} \geq \boxed{\lambda/(4\pi)}$$ Natural emittance of light

Fundamentals of Light and SR

- Why we need SR?
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.

- The characteristics of SR is thus quite different due to relativistic effects.

- What we have to take care is:

    1. Speed-of-light limit
    2. Squeezing of light pulse
    3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of an electron never exceeds the speed of light.

$$v/c = \beta = \sqrt{1 - \gamma^{-2}}$$

$$\sim 1 - \frac{1}{2\gamma^2}$$
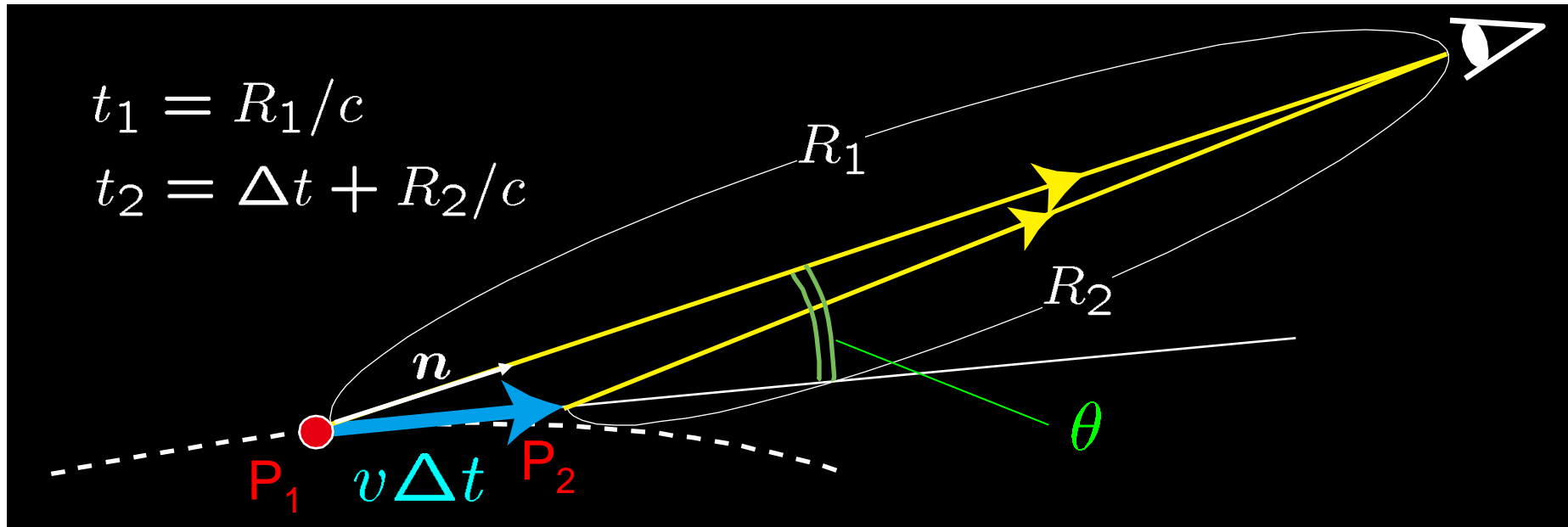
$$\gamma = \frac{E}{mc^2}$$

Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

:Lorentz Factor
(relative electron energy,mc^2=0.511MeV)

Squeezing of Light Pulse Duration

$$t_1 = R_1/c$$
$$t_2 = \Delta t + R_2/c$$

R_1

R_2

n

$v\Delta t$

P_1 P_2

θ

$$R_2 = \sqrt{(R_1)^2 + (v\Delta t)^2 - 2R_1 v\Delta t \cos\theta}$$
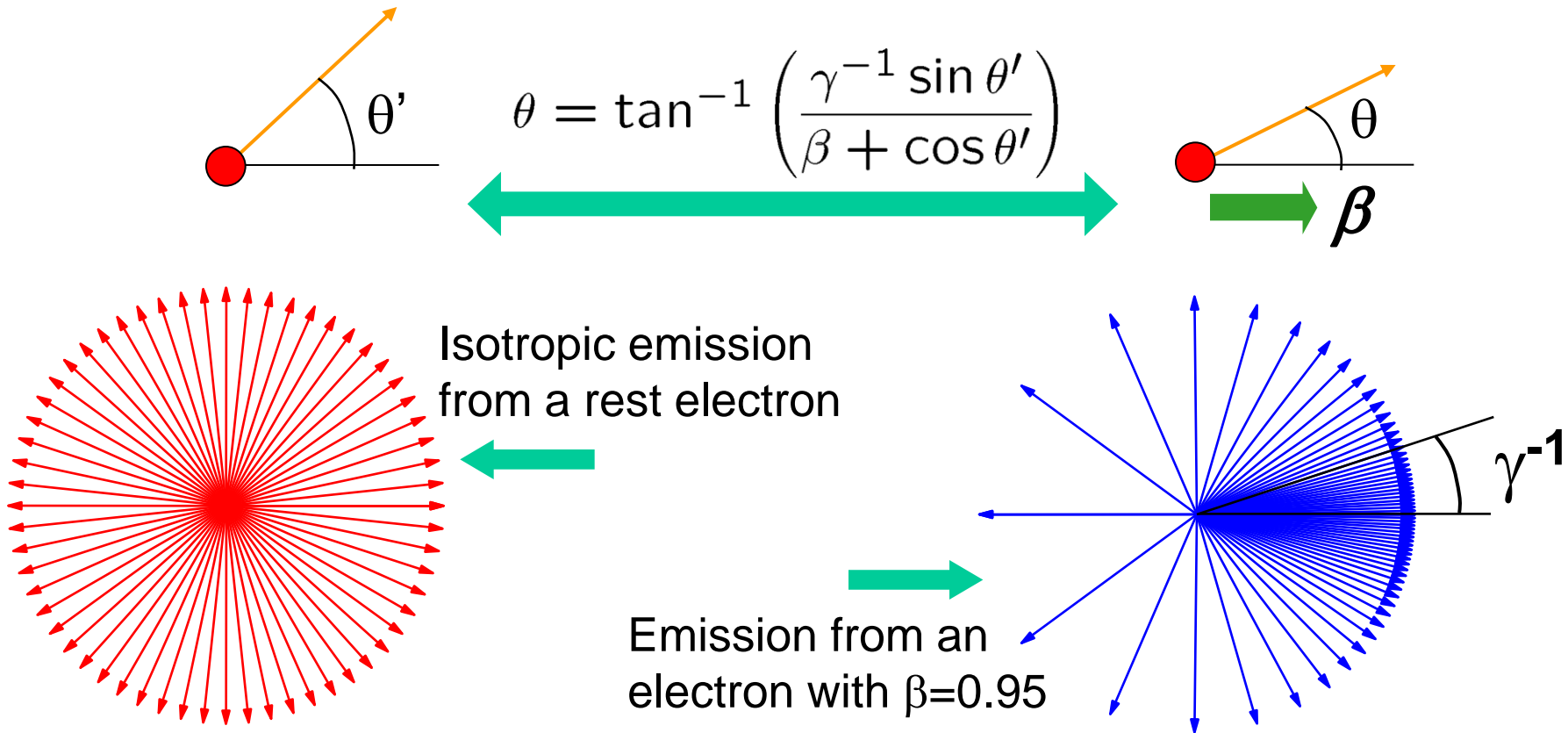$$\sim R_1 - (\boldsymbol{v} \cdot \boldsymbol{n})\Delta t$$
$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$
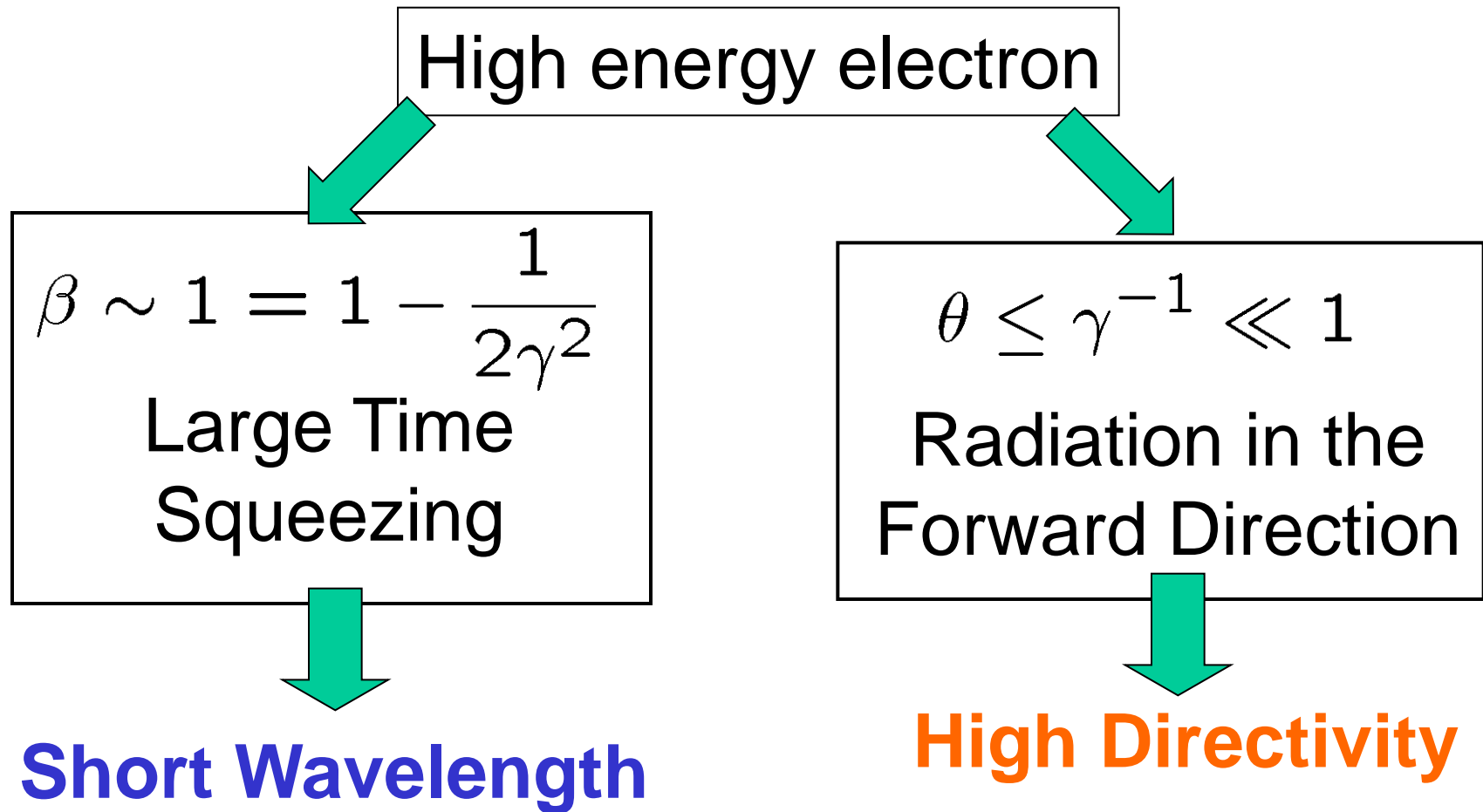$$= \Delta t \boxed{(1 - \boldsymbol{\beta} \cdot \boldsymbol{n})} = \boxed{\frac{\Delta t}{2\gamma^2}} \quad \gamma \gg 1, \theta = 0$$

time squeezing

Conversion of Emission Angles

$$\theta = \tan^{-1}\left(\frac{\gamma^{-1}\sin\theta'}{\beta + \cos\theta'}\right)$$

θ'

θ

β

Isotropic emission from a rest electron

Emission from an electron with $\beta = 0.95$

γ^{-1}

Light emitted from a moving object ($\beta \sim 1$) concentrates within γ^{-1}

SR from a High-Energy Electron

High energy electron

$$\beta \sim 1 = 1 - \frac{1}{2\gamma^2}$$

Large Time Squeezing

$$\theta \leq \gamma^{-1} \ll 1$$

Radiation in the Forward Direction

Short Wavelength

High Directivity

Overview of SR Light Source

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.

Bending Magnet (BM)

Insertion Device (ID)
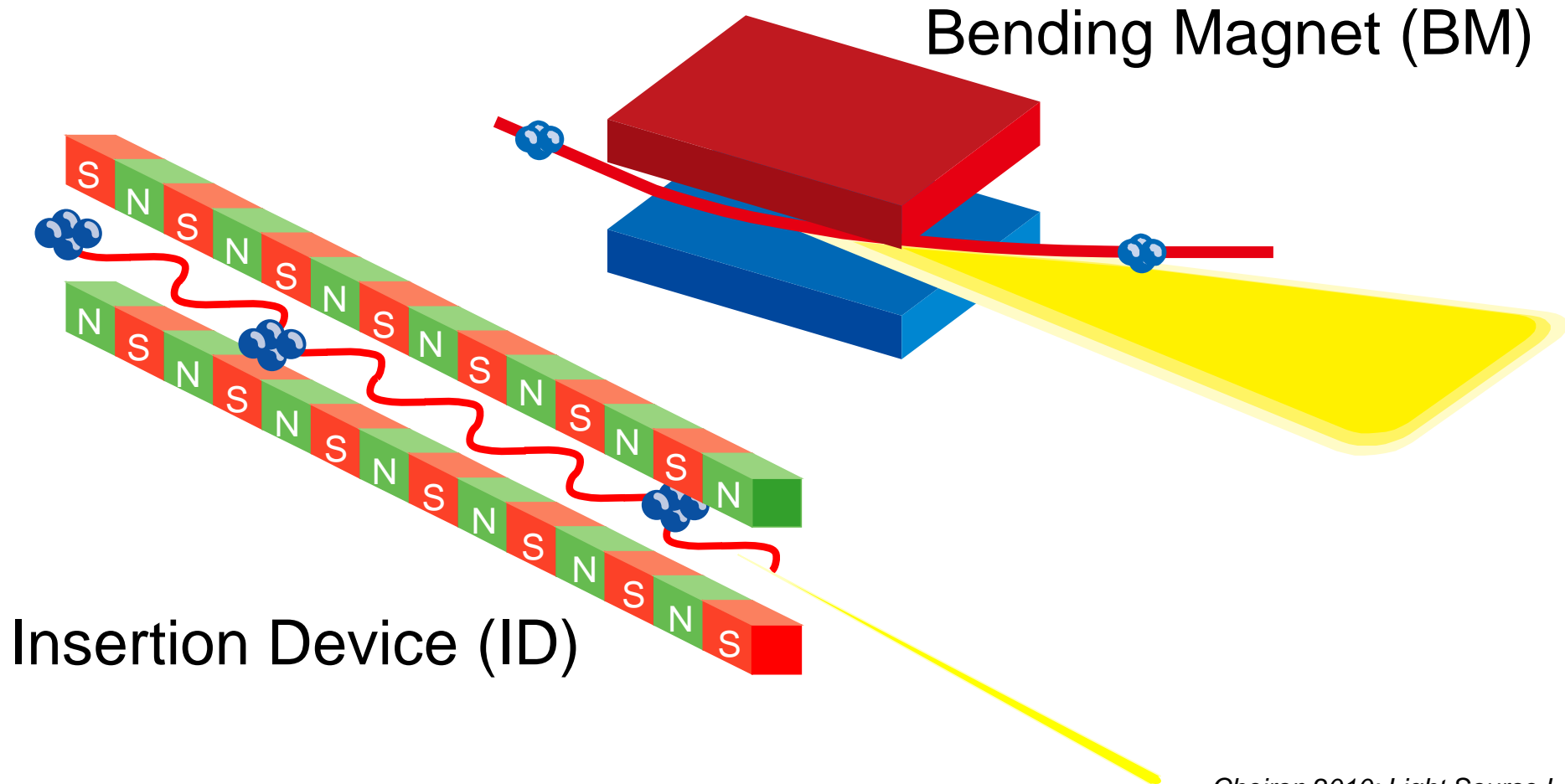
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- EMs combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.

- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.

- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.

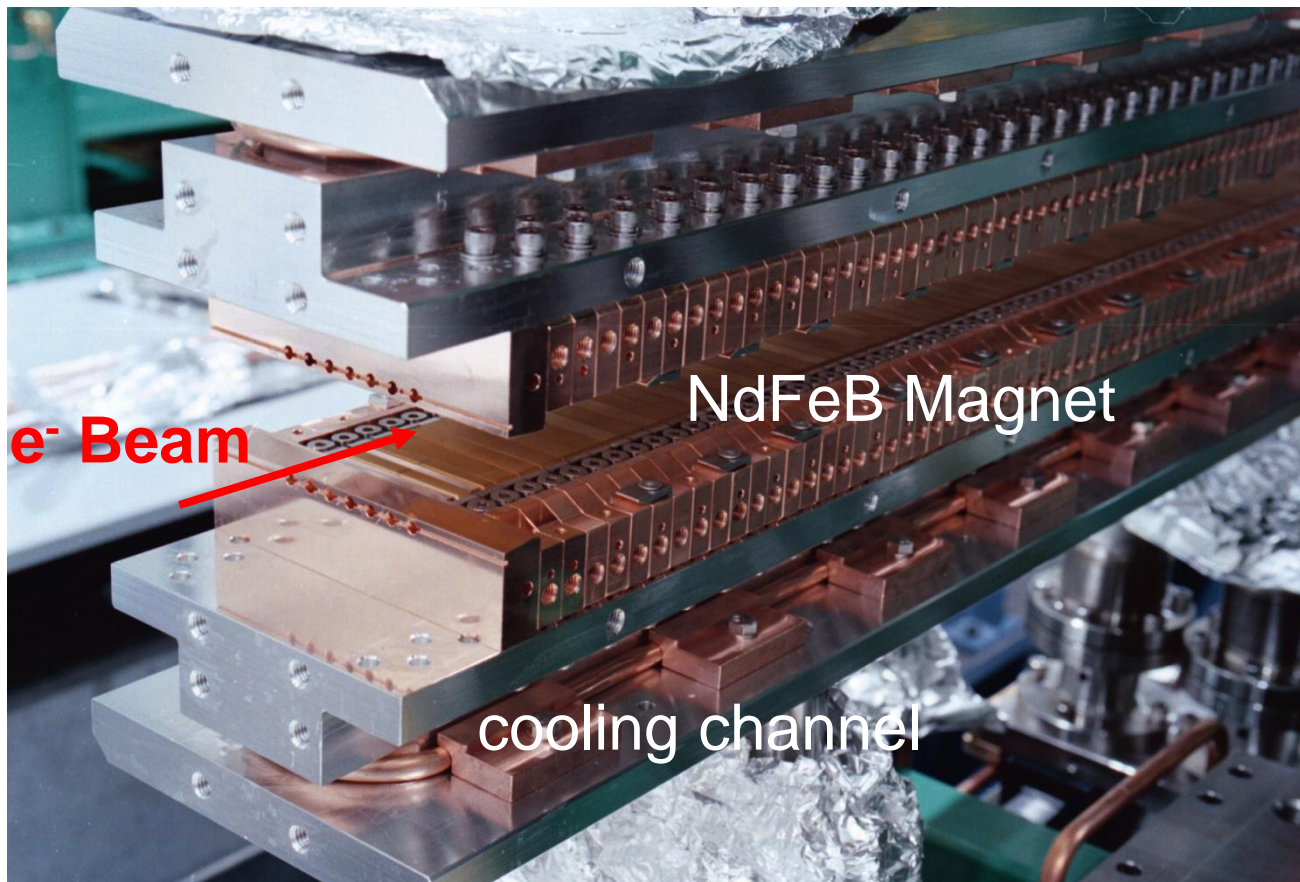- Classified into wigglers and undulators.

Magnetic Circuit of IDs

Halbach type

Hybrid type

pole piece
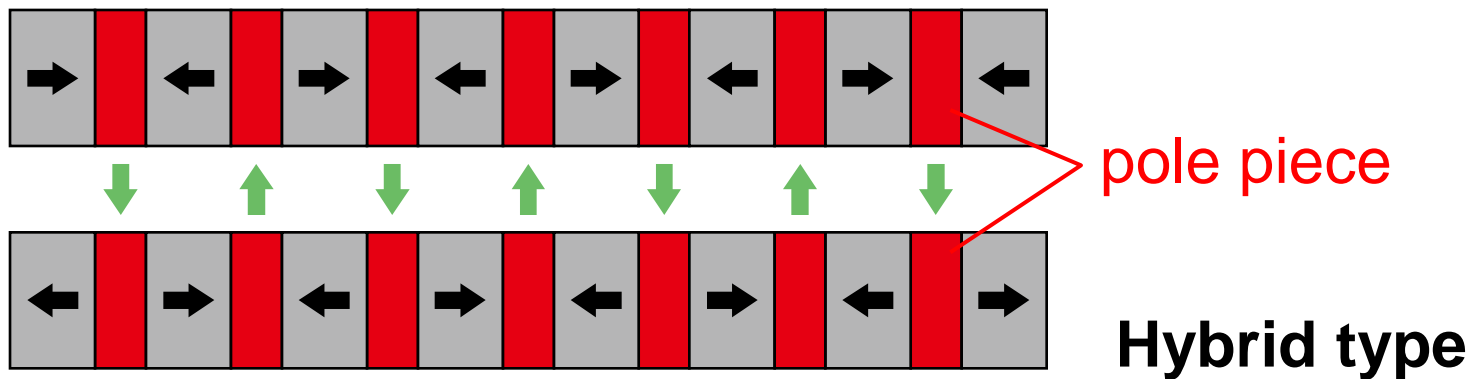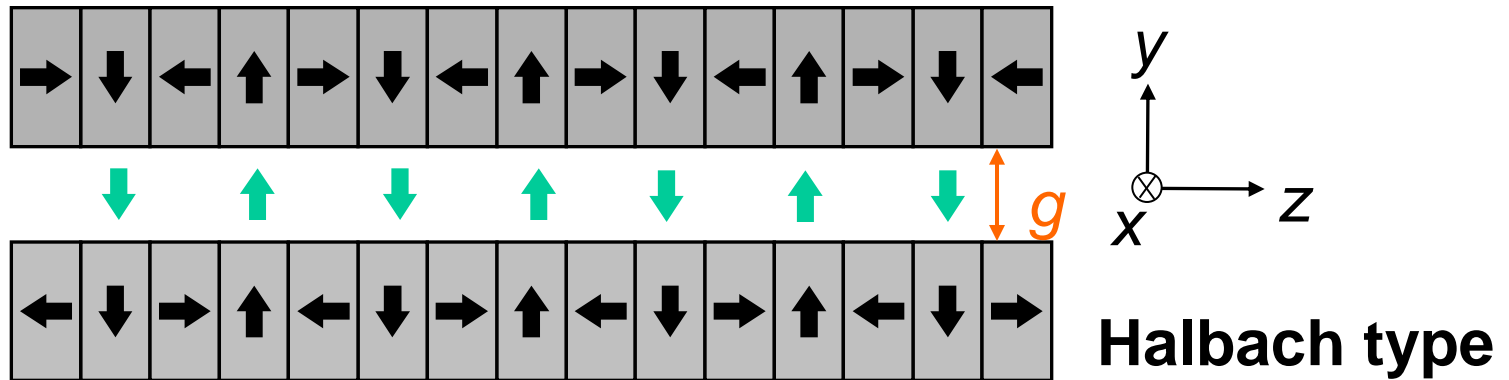
In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u) \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

Example of ID Magnets

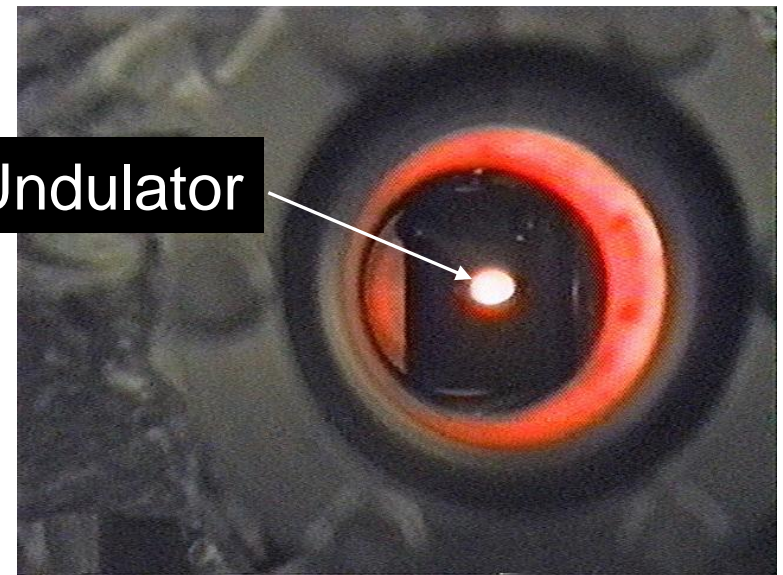## Halbach-type Magnet Array for SPring-8 Standard Undulators

e$^-$ Beam

NdFeB Magnet

cooling channel

Example of SR Image

BL41XU@SP-8, First Image of SR at Fluorescent Screen (<0.1mA)

SR from Downstream BM

SR from Undulator

SR from Upstream BM
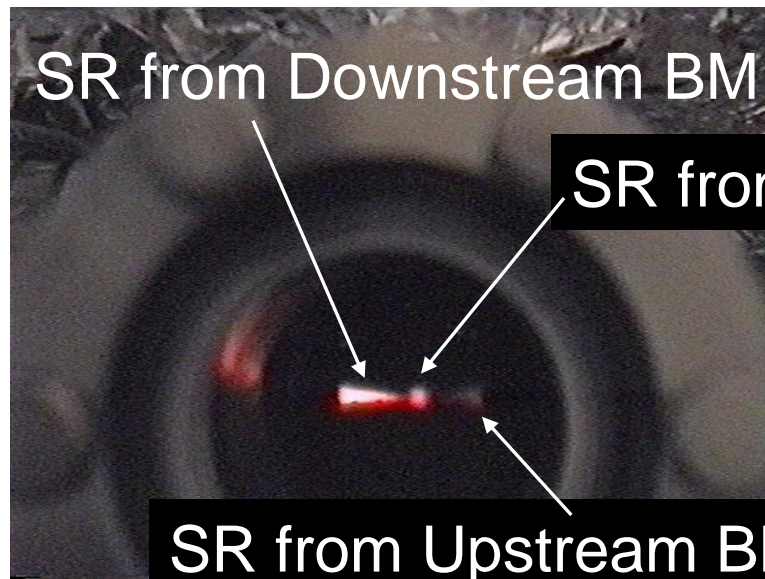
Undulator Gap = 50 mm

Undulator Gap = 20 mm

Comparison of Light Sources
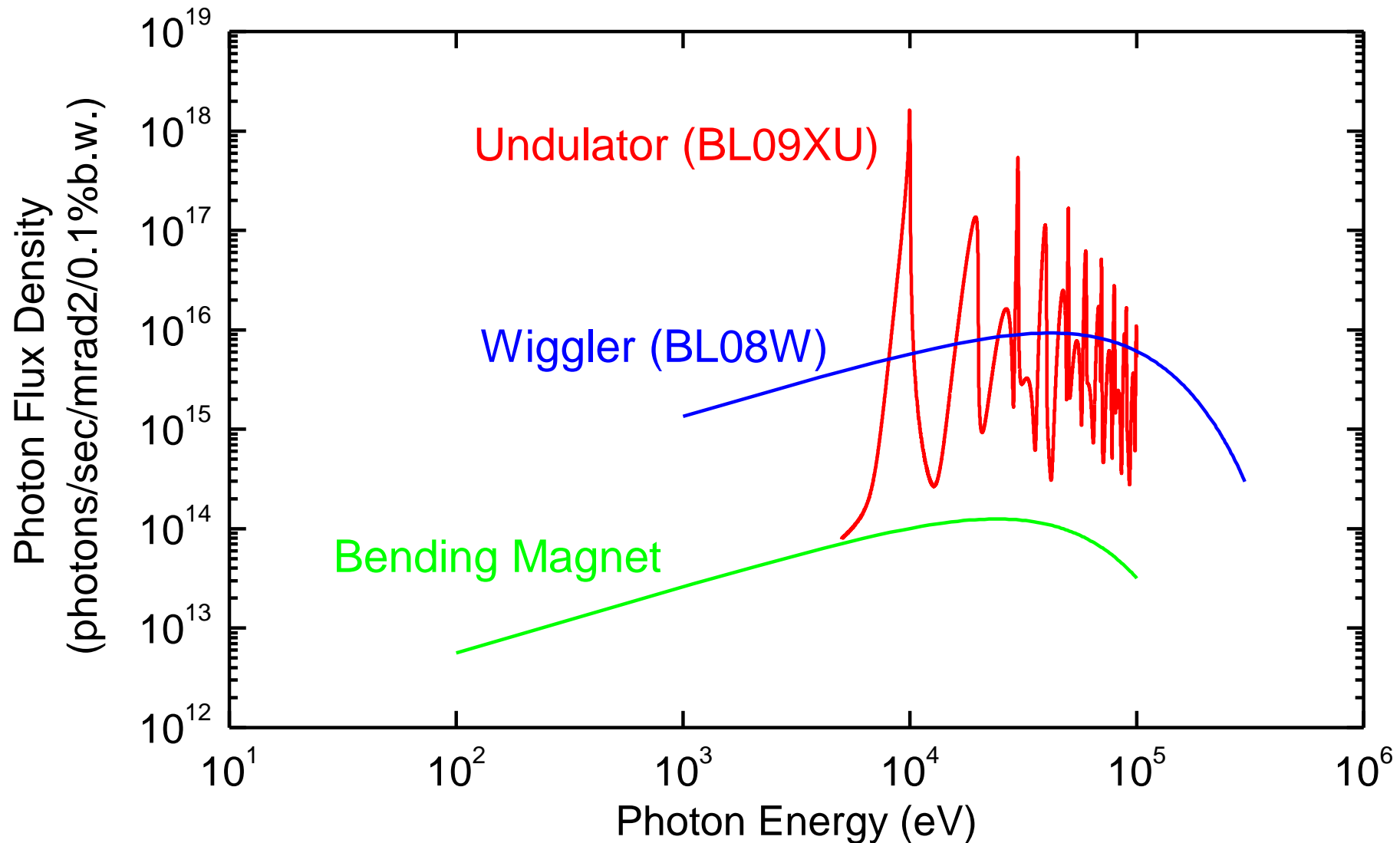
Characteristics of SR (1)
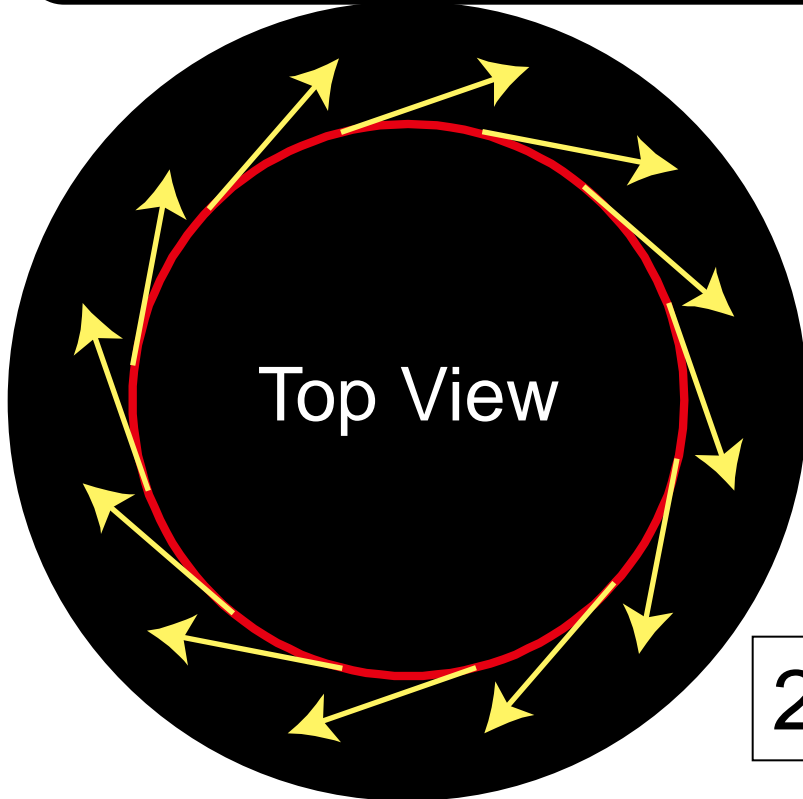
- Radiation from BMs

Directivity of BM Radiation

Side View

γ^{-1}
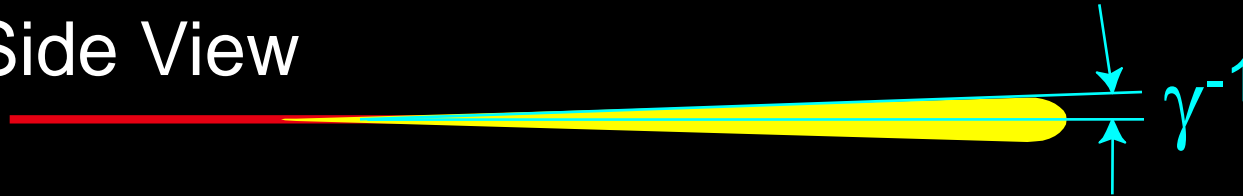
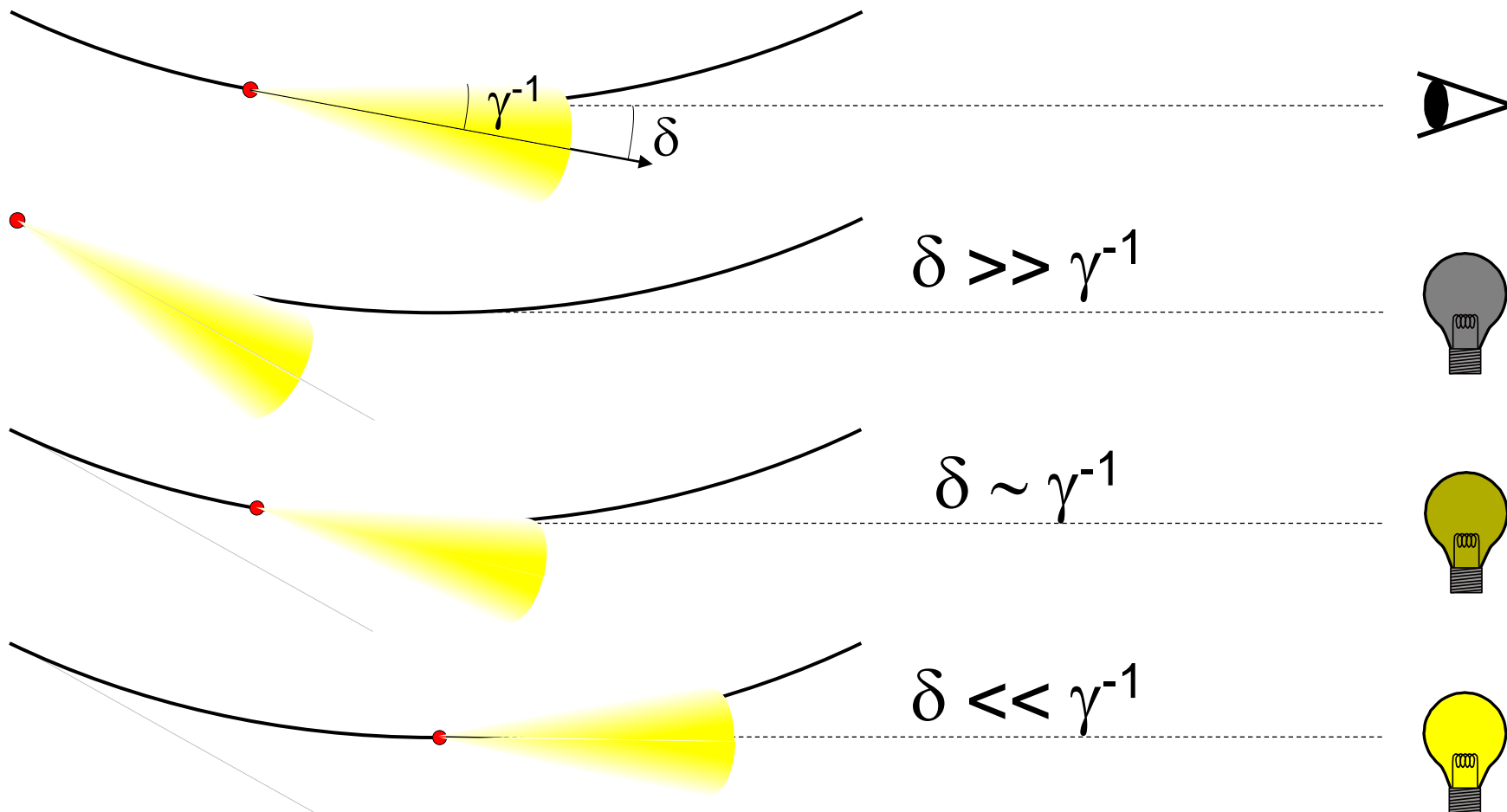High directivity in the vertical plane ($\sigma_{y'} \sim \gamma^{-1} \sim 64\mu$rad@SP-8)

Top View

Isotropic in the orbital plane

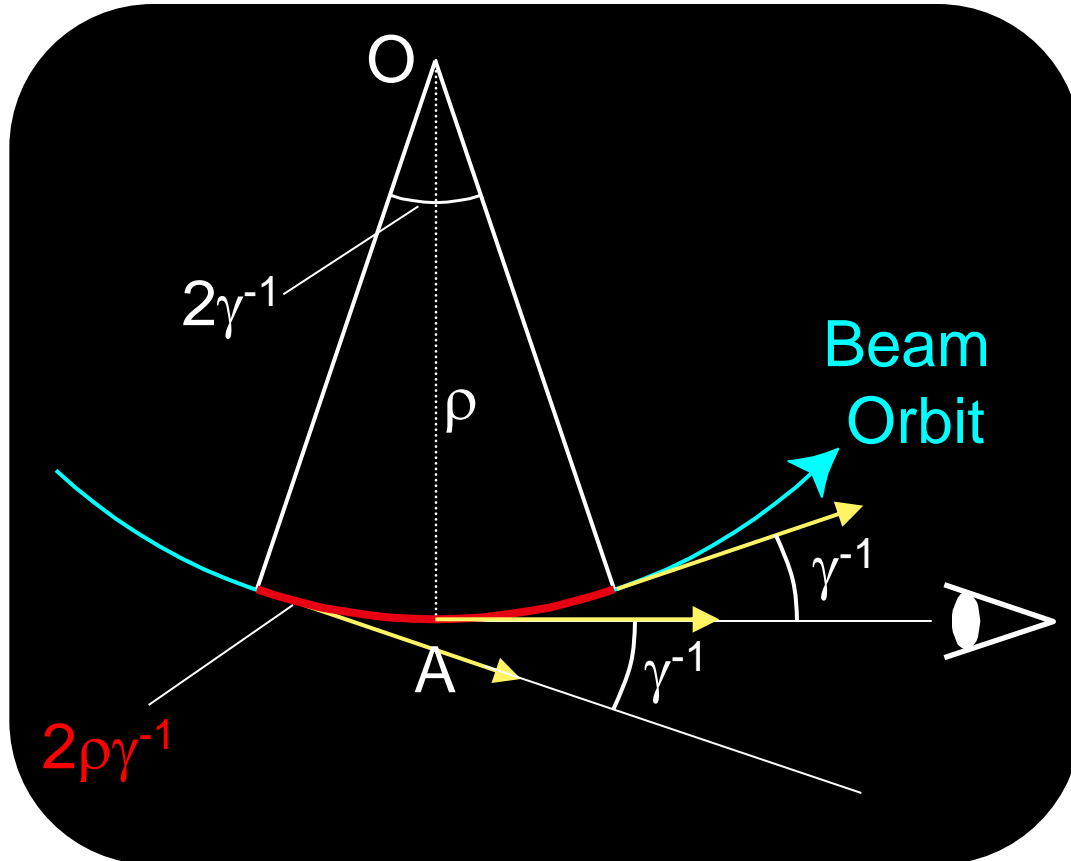2-dimensional directivity

Spectrum of BM Radiation (1)

γ^{-1}

δ

$\delta \gg \gamma^{-1}$

$\delta \sim \gamma^{-1}$

$\delta \ll \gamma^{-1}$

Photons emitted when $\delta < \gamma^{-1}$ are detected by the observer

Spectrum of BM Radiation (2)

Major contribution of radiation is from the portion painted red
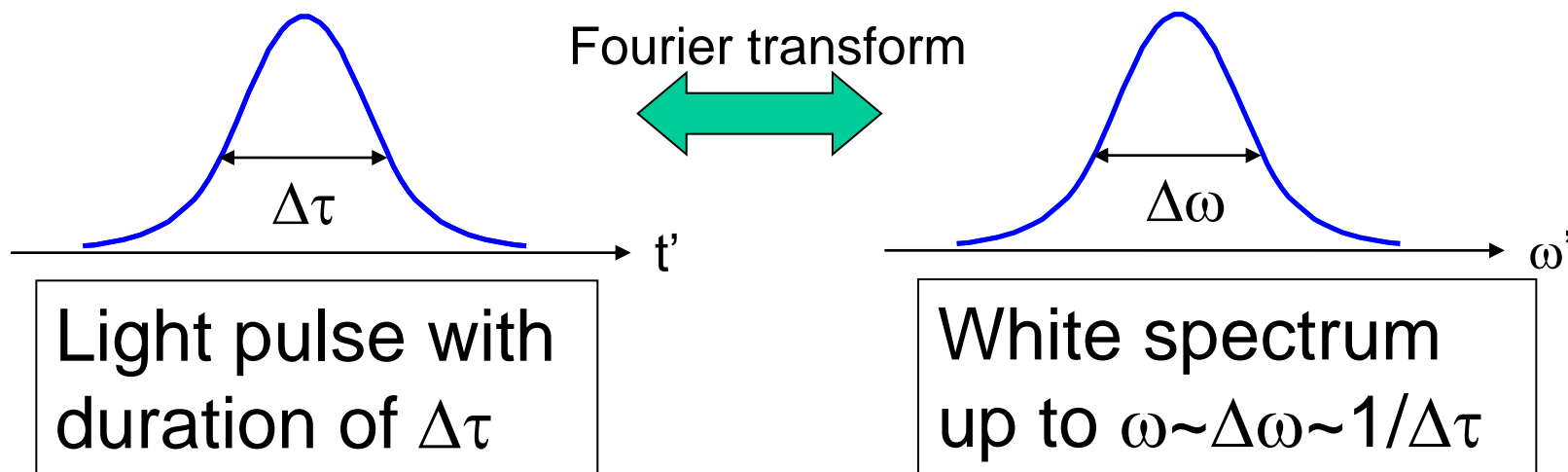
Pulse duration for e⁻
$$\Delta t = 2\rho\gamma^{-1}/c$$

squeezing

Pulse duration for observer
$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$
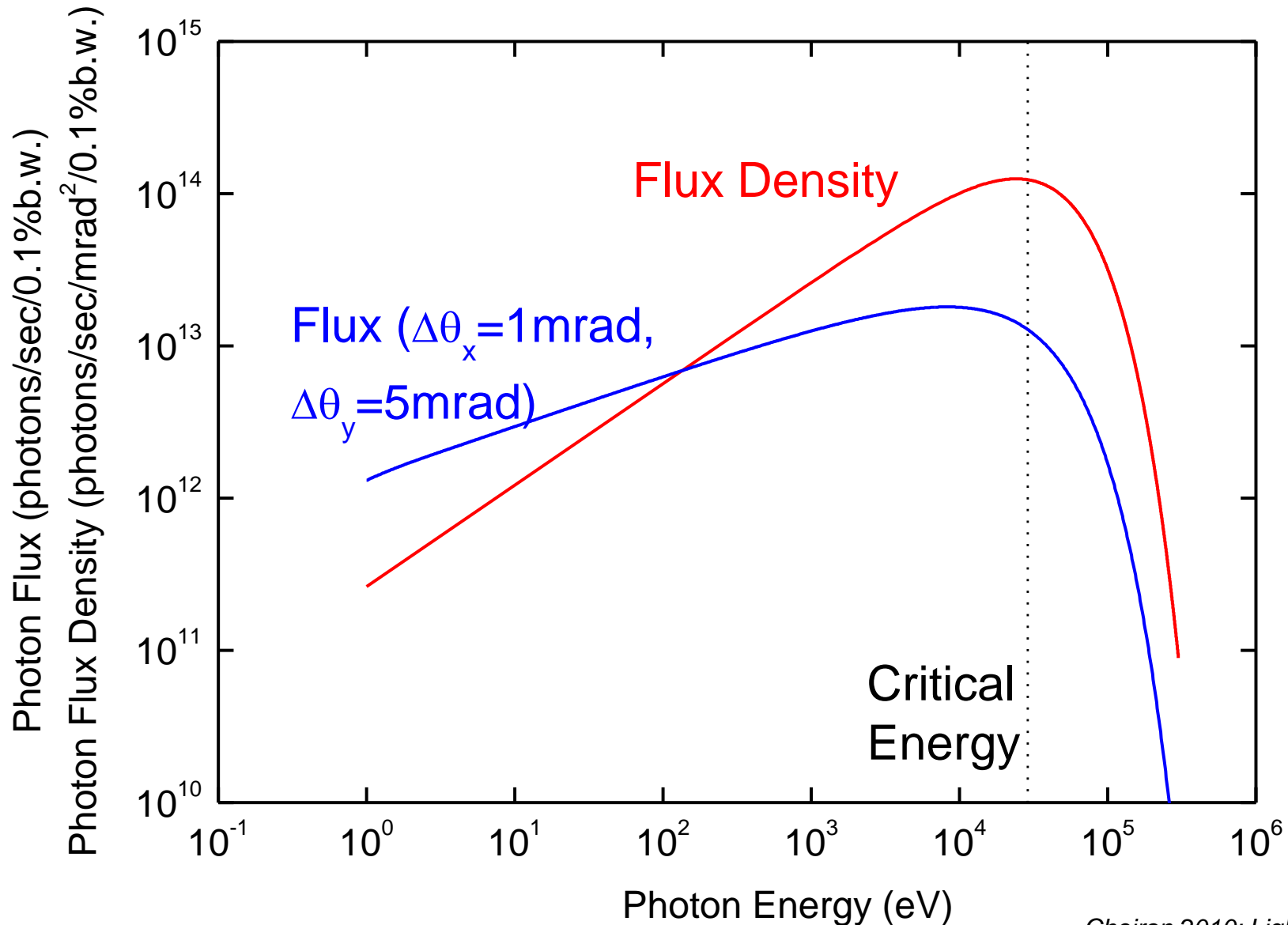
Spectrum of BM Radiation (3)

Fourier transform

$\Delta\tau$

t'

$\Delta\omega$

ω'

Light pulse with duration of $\Delta\tau$

White spectrum up to $\omega\sim\Delta\omega\sim1/\Delta\tau$

- By definition, $\omega_c=(3/2)/\Delta\tau=3\gamma^3c/2\rho$ is called "critical frequency" of SR, which gives a criterion of the maximum energy of SR from a BM.
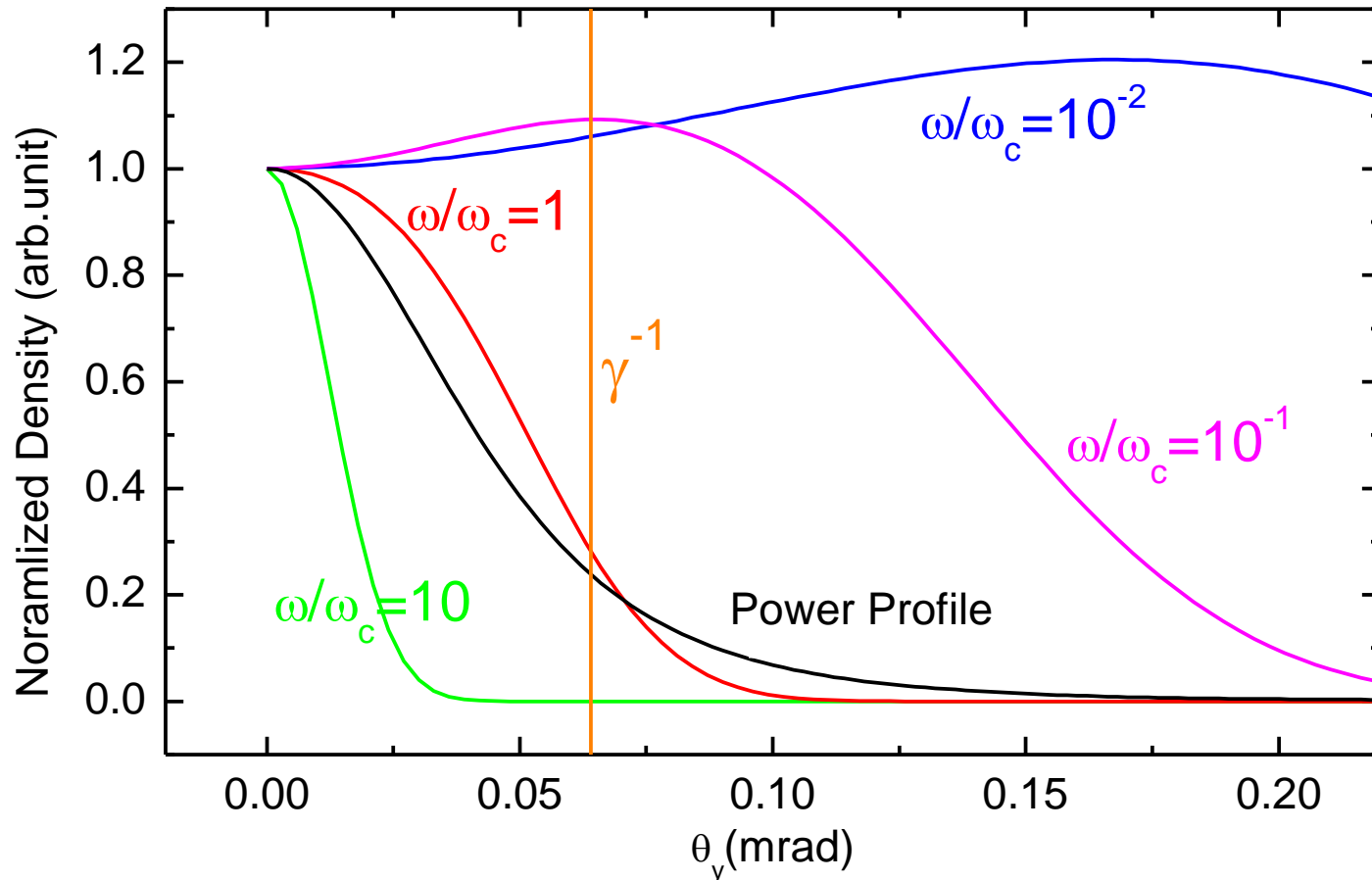
- In practical units,
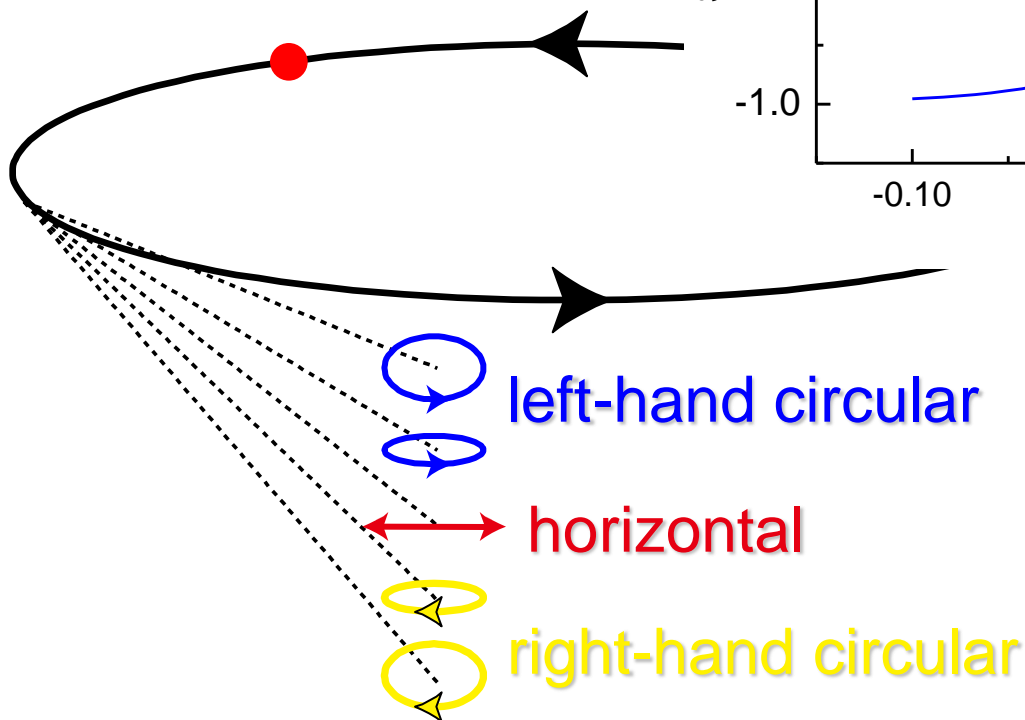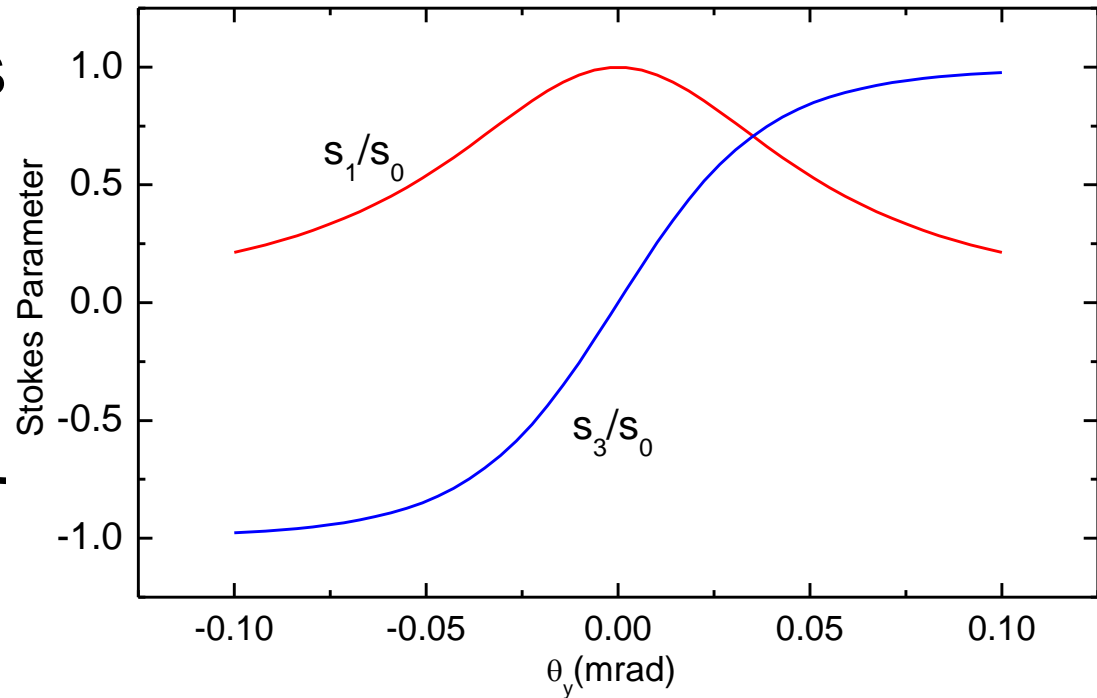$$\hbar\omega_c(keV)=0.665E_e^2(GeV)B(T)$$

Example of Spectrum

Angular Profile of BM Radiation

- power profile ~ flux profile @ $\omega/\omega_c=1$
- larger angular divergence for lower energy

Polarization of BM Radiation

Stokes parameters of BM radiation along vertical axis

left-hand circular

horizontal

right-hand circular

Polarization state reflects the apparent motion of electron.