

# Ring Accelerator Physics for Non Accelerator Specialists

Hitoshi TANAKA  
XFEL Division  
JASRI

# Outline

1. Introduction
2. Light source properties vs electron beam performance
3. Stochasticity of photoemission
4. Distribution of circulating electron beam
5. Extra: Approach to coherent X-rays

# 1. Introduction (1)

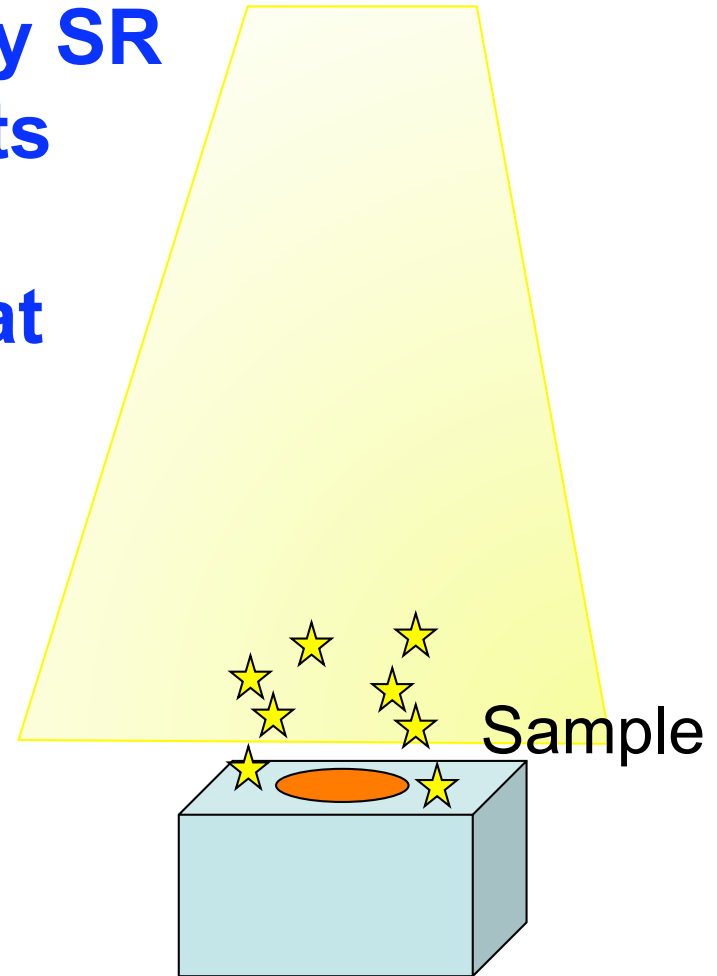
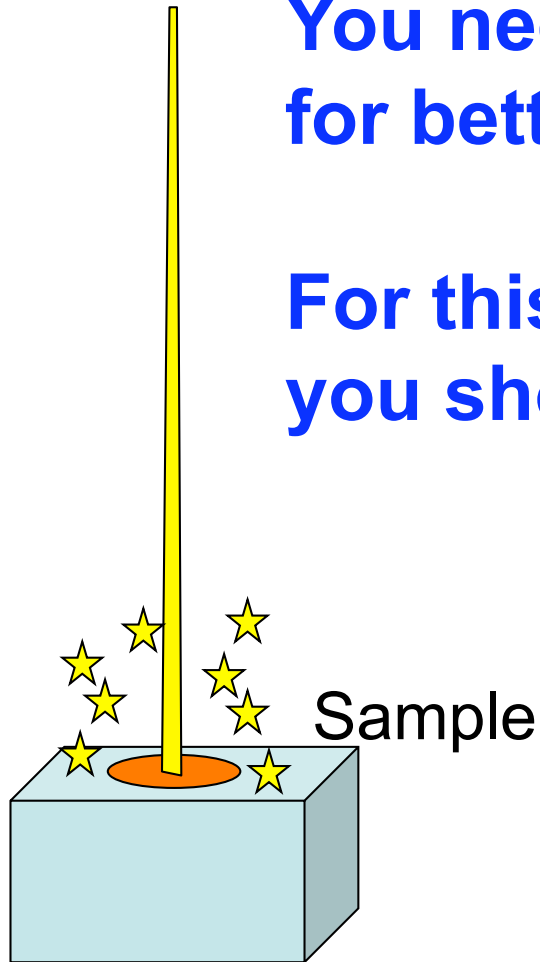
Even any member in an experiment-related division or BL scientists, who are not the accelerator specialists, should understand the relation between the SR properties and electron beam performances,

**to keep** the present experimental condition,  
**to improve** the condition as much as possible,  
and  
**to challenge** new experiments by using  
the advanced SR.

# 1. Introduction (2)

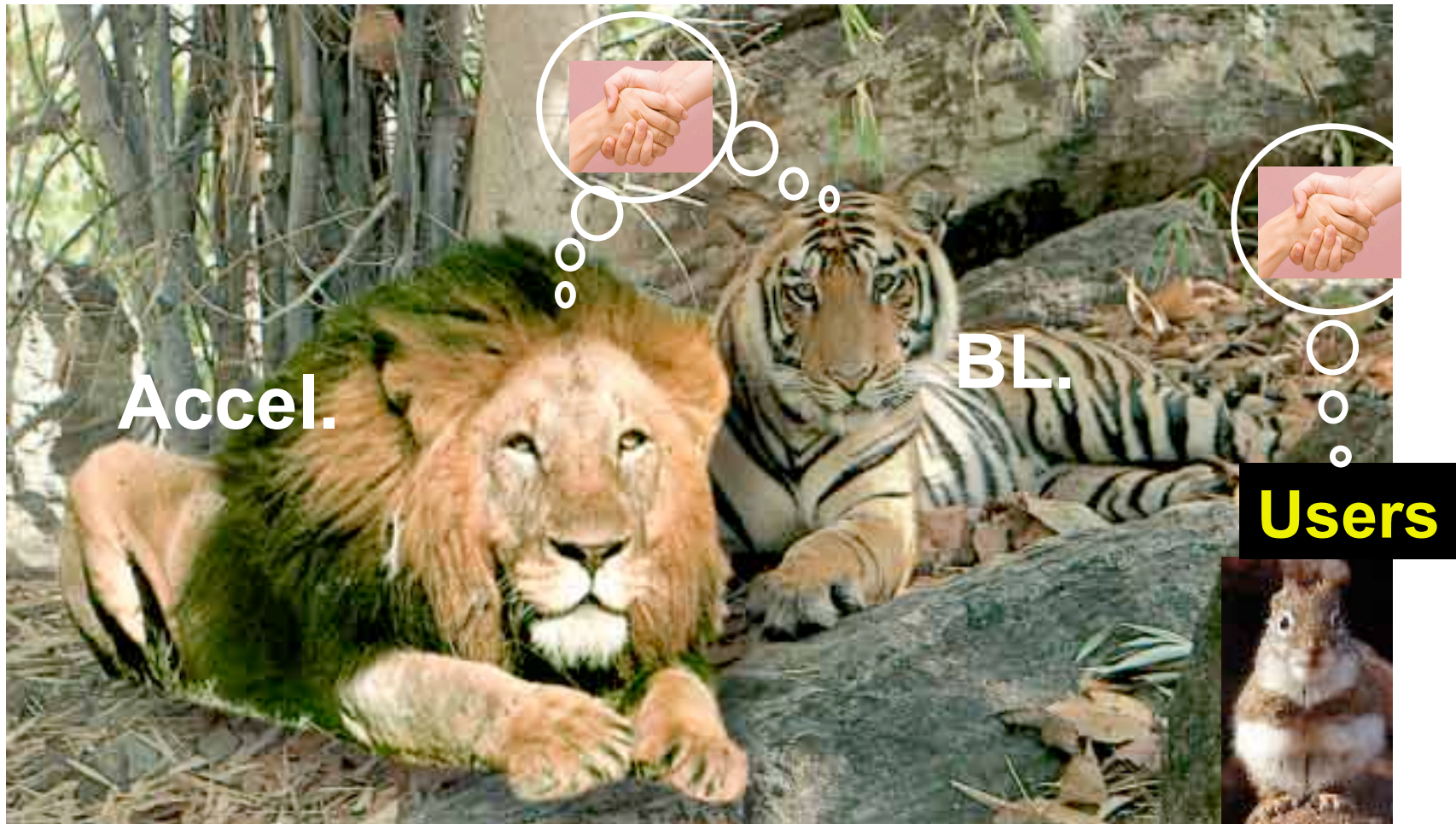
**You need high quality SR  
for better experiments**

**For this purpose what  
you should do?**



# 1. Introduction (3)

## Mutual understanding through communication

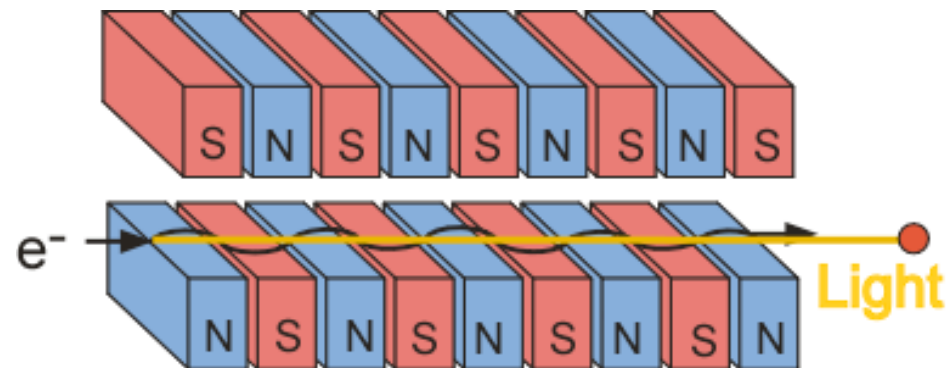
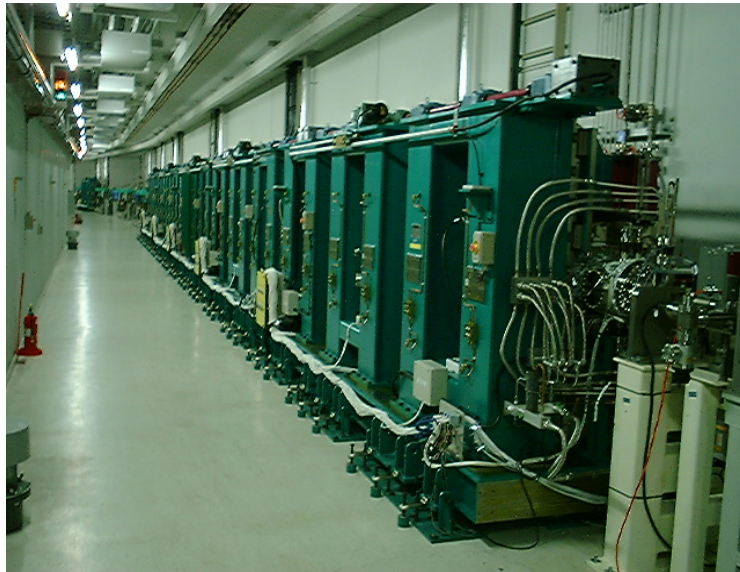


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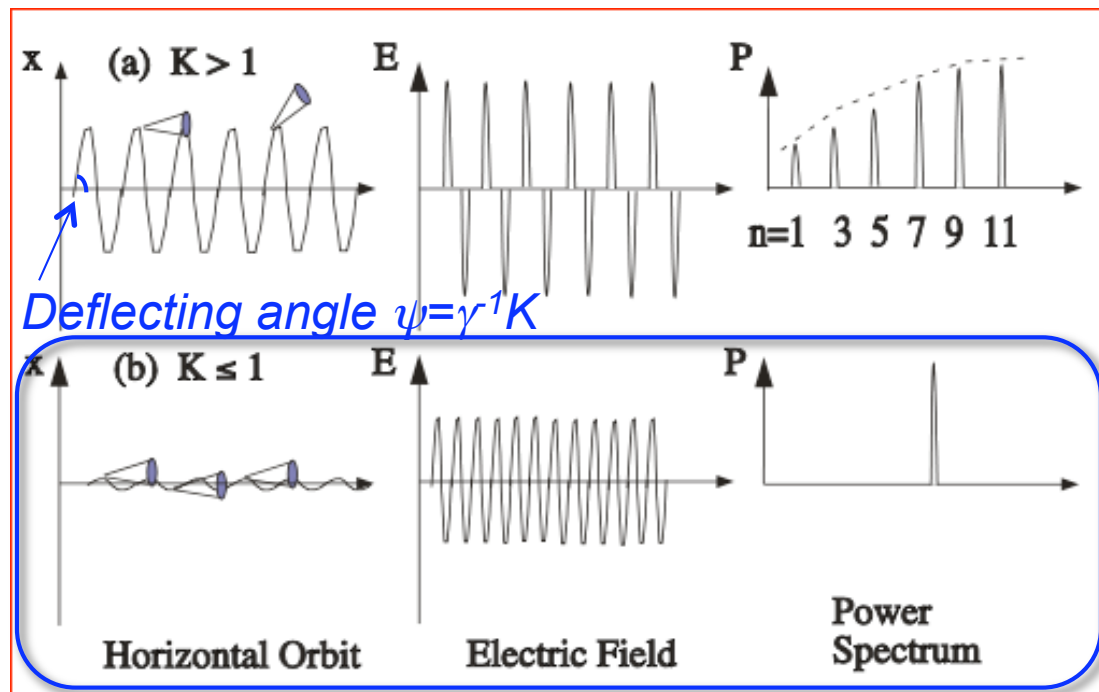
## 2. Light source properties vs electron beam performance(1)

Main devices to supply SR to users are “undulators”, which are installed in magnet-free straight sections in 3rd-generation SR sources.



## 2. Light source properties vs electron beam performance(2)

In an undulator, radiation from a single electron at each undulation interferes with each other.



Spectral narrowing by resonant condition for a single electron is

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2} (1 + K^2/2 + \gamma^2\theta^2)$$

where  $K$  and  $\gamma$  are a deflecting parameter and relative energy.

## 2. Light source properties vs electron beam performance(3)

Q: Why does the radiation concentrate within the  $\gamma$  cone ?

A: The electron can run after the emitted light with the almost light speed only in this limited angle.

retarded time

present time

$$t = t' + \frac{R(t')}{c}$$

Time squeezing factor  $\kappa(t') = \frac{\Delta t}{\Delta t'}$

$\kappa(t')$  small  $\rightarrow$  large acceleration  $\rightarrow$  strong radiation

$$\kappa(t') = \frac{\Delta t}{\Delta t'} = 1 - \vec{n}(t') \cdot \vec{\beta}(t')$$

$$\kappa(t') \approx \frac{1}{2} \left( \frac{1}{\gamma^2} + \theta^2 \right)$$

Origin

$\vec{z}(t')$

$\vec{\beta}(t')$

$\vec{x}$

$\theta(t')$

$\vec{n}(t')$

$R(t') = |\vec{x} - \vec{z}(t')|$

$P(\vec{x})$

Observation Point

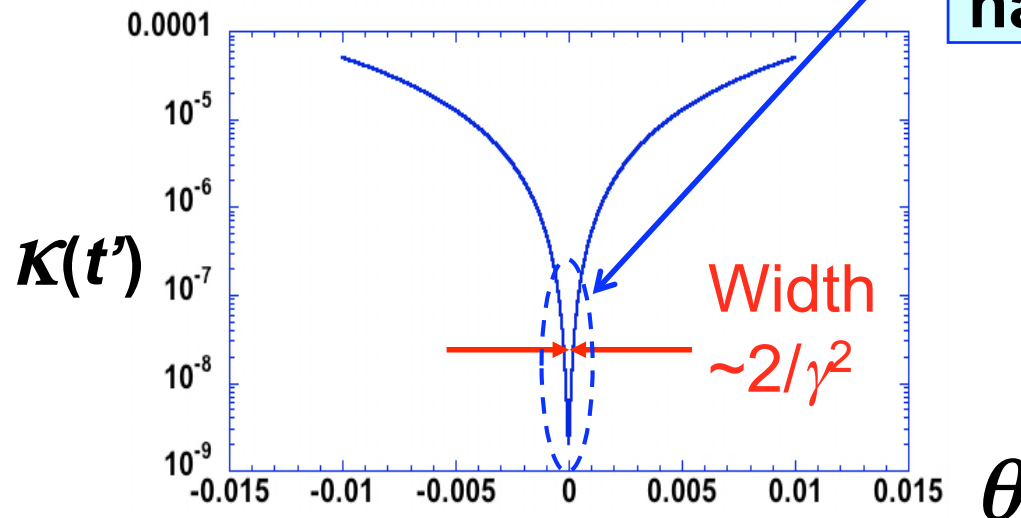
## 2. Light source properties vs electron beam performance(4)

Q: Why does the radiation concentrate within the  $\gamma$  cone ?

A: The electron runs after the emitted light with the almost light speed only in this limited angle.

$$K(t') \approx \frac{1}{2} \left( \frac{1}{\gamma^2} + \theta^2 \right) \quad \theta < 1/\gamma \rightarrow K \sim 1/\gamma^2$$

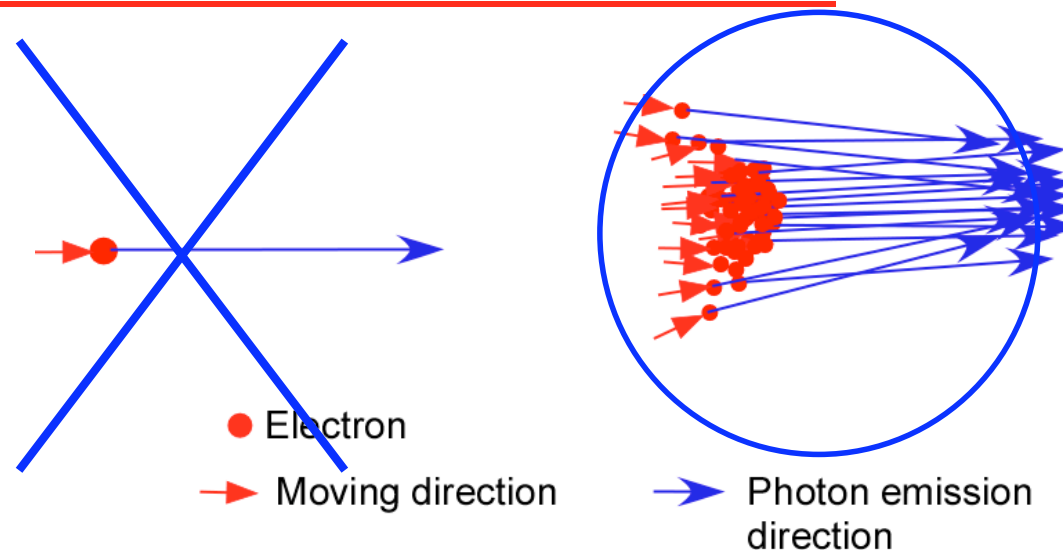
**Intense radiation is emitted in this narrow angle**



## 2. Light source properties vs electron beam performance(5)

If electron beam were point-like without spatial divergence and , all electrons could have no angular divergence, radiations were **coherent** !

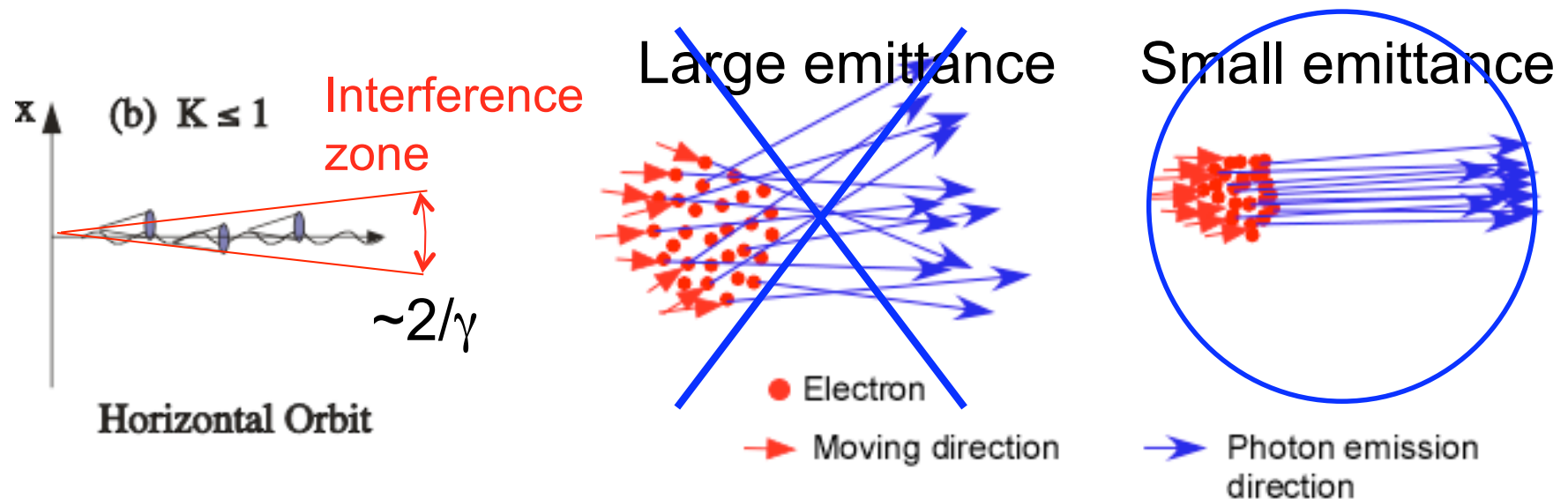
**Real radiation properties are obtained by convoluting radiations from all N electrons**



In the real world N electrons are distributed in a phase space, never degenerate on the same point.

## 2. Light source properties vs electron beam performance(6)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently **small beam emittance** is required.



## 2. Light source properties vs electron beam performance(7)

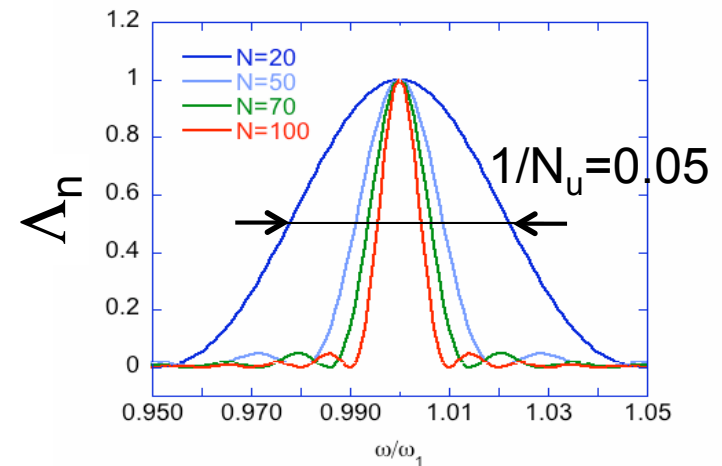
In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently **small beam energy spread** is required.

$$\lambda_1 = \frac{\lambda_u}{2(\gamma + \Delta\gamma)^2} (1 + K^2/2 + \gamma^2\theta^2)$$

$$\Delta\lambda_1 \sim 2\lambda_1 \langle \Delta\gamma/\gamma \rangle$$

Spectral broadening by the energy spread is much less than intrinsic broadening  $\sim 1/N_u$ .

$N_u$ : undulator period number



## 2. Light source properties vs electron beam performance(8)

$$B \propto \frac{I_b \cdot N_u^2 \cdot \gamma^2 \cdot F_n(K, \delta)}{\sigma_x \sigma_{x'} \sigma_y \sigma_{y'} (\sim \epsilon_x \cdot \epsilon_y)},$$

B: Brilliance (phs/sec/mm<sup>2</sup>/mrad<sup>2</sup>/100mA)

I<sub>b</sub>: Beam current (mA)

N<sub>u</sub>: Undulator period number

σ<sub>x</sub>, σ<sub>y</sub>: Horizontal and vertical beam sizes (m)

σ<sub>x'</sub>, σ<sub>y'</sub>: Horizontal and vertical angular divergence (rad)

δ: Beam energy spread

K :Deflection parameter

## 2. Light source properties vs electron beam performance(9)

$$\sigma_x = \sqrt{\sigma_p^2 + \beta_x \varepsilon_x + \eta_x^2 \delta^2}, \quad \sigma_{x'} = \sqrt{\sigma_{p'}^2 + \gamma_x \varepsilon_x + \eta'_x{}^2 \delta^2},$$

$$\sigma_y = \sqrt{\sigma_p^2 + \beta_y \varepsilon_y + \eta_y^2 \delta^2}, \quad \sigma_{y'} = \sqrt{\sigma_{p'}^2 + \gamma_y \varepsilon_y + \eta'_y{}^2 \delta^2},$$

$$\alpha_{x,y} = \frac{-d\beta_{x,y}}{ds}, \quad \gamma_{x,y} = \frac{1 + \alpha_{x,y}^2}{\beta_{x,y}}, \quad \eta'_{x,y} = \frac{d\eta_{x,y}}{ds}.$$

**Photon**

**Electron  
emittance**

**Electron  
energy  
spread**

$\varepsilon_x, \varepsilon_y$ : Horizontal and vertical emittance (m•rad)

$\beta_{x,y}$ : Horizontal and vertical betatron functions at ID

$\eta_{x,y}$ : Horizontal and vertical dispersion functions at ID

$\sigma_p, \sigma_{p'}$ : Spatial and angular divergence of photon beam

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### 3. Stochasticity of photoemission(1)

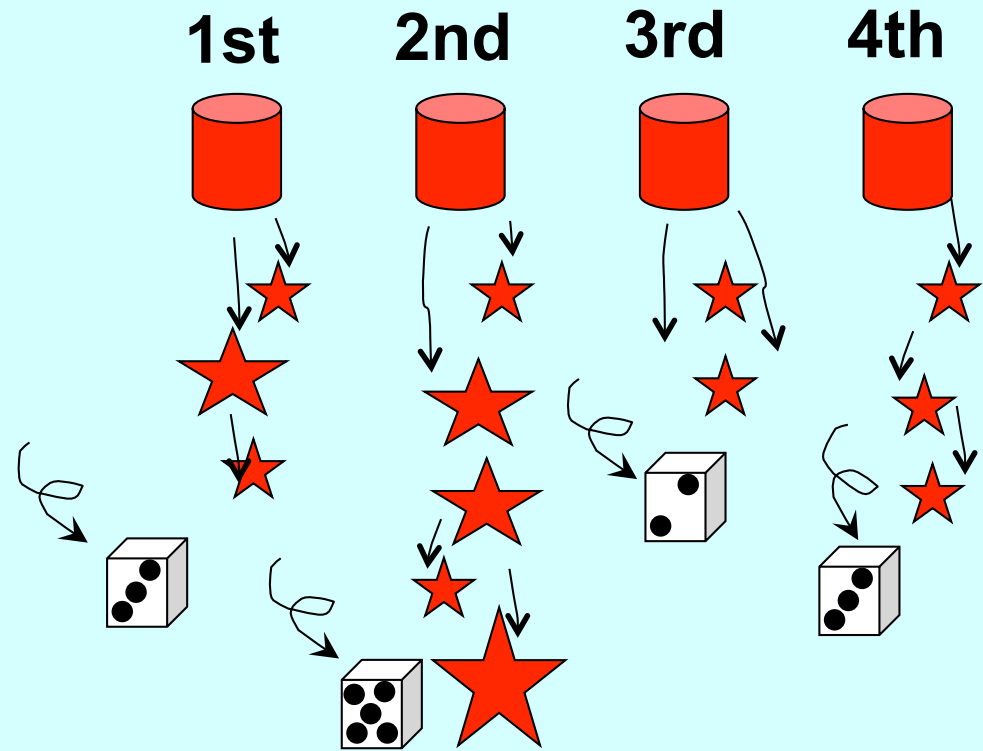
The photo-emission process is **not continuous**, in other words, **quantized process**. So, the emission position, the number of the emission photons, and the emission photon energy have **fluctuations**.

*This stochasticity (random fluctuation) causes finite spread of the circulating electron beam in the 6D phase space. The density distribution is generally Gaussian due to the central limit theorem.*

# 3. Stochasticity of photoemission(2)



**Continuous fluid**



**Photoemission process**

### 3. Stochasticity of photoemission(3)

A relativistic electron accelerated in a magnetic field will radiate electromagnetic energy at a rate which is proportional to the square of the accelerating force.

<Averaged radiation power>

$$P = \frac{2}{3} \frac{r_e c}{(m_0 c^2)^3} E^2 F_{\perp}^2 = \frac{2}{3} \frac{r_e c}{(m_0 c^2)^3} \frac{E^4}{\rho^2} .$$

classical electron radius

<Averaged radiation energy per turn>

$$U = \int P dt = \int P \frac{dl}{c} ,$$

Averaged properties are smooth !

$$U(\text{keV}) = \frac{88.5 E^4 (\text{GeV})}{\rho(\text{m})} \text{ for the case with the constant } \rho .$$

### 3. Stochasticity of photoemission(4)

In the quantized radiation process, the integrated parameter, energy loss per unit time also fluctuates. Magnitude of the fluctuation can be defined by a mean square.

<Mean square of energy loss fluctuation per unit time>

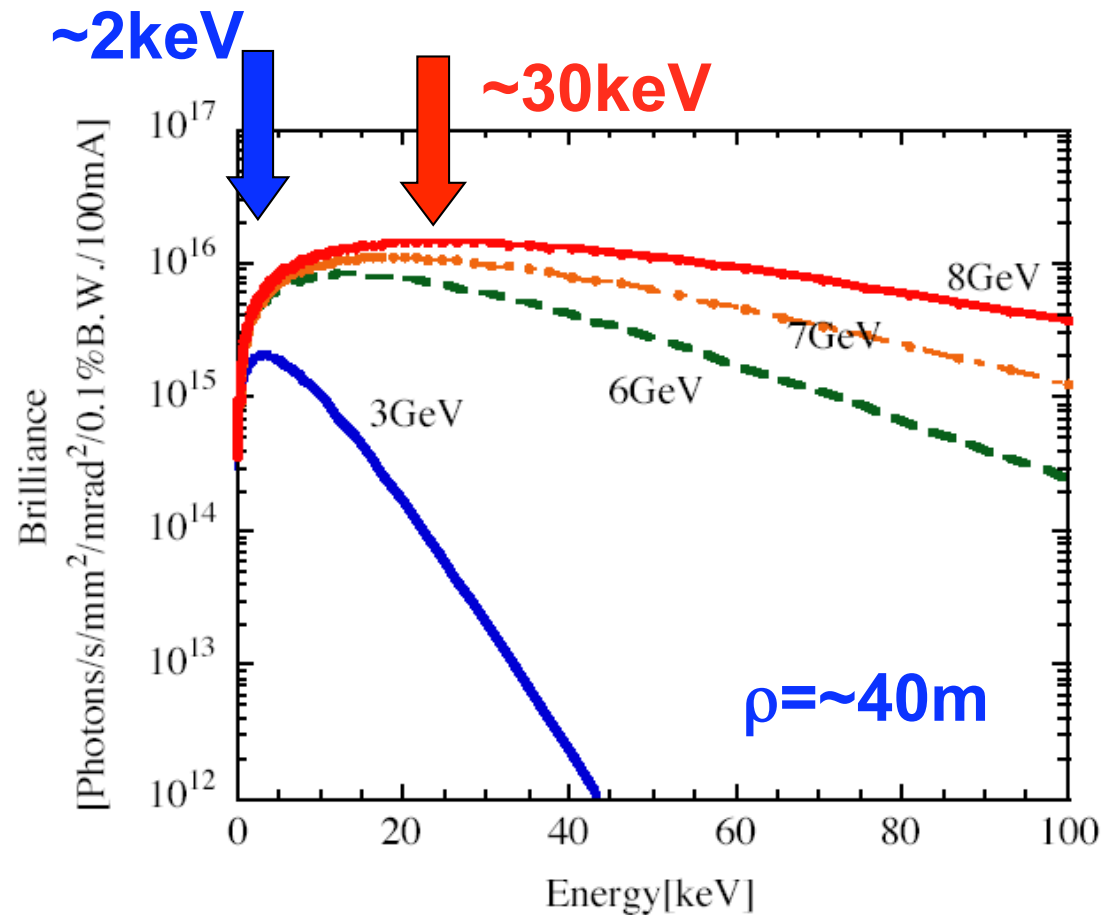
$$\langle Nu^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c, \quad u_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3.$$

$u_c$ : critical photon energy  $\sim 3.2\langle u \rangle$

<Averaged photo-emission rate>

$$\langle N \rangle (\text{phs/s}) \sim 3.2P / u_c$$

### 3. Stochasticity of photoemission(5)



$U_c$  represents the photon energy at the peak brilliance of the BM radiation.

### 3. Stochasticity of photoemission(6)

Let's estimate the parameters!

<Case-1 E=1 GeV,  $\rho=5m$ >

$$U(\text{keV}) = \frac{88.5 \times 10^4}{5} = 17.7, \quad P(\text{keV/s}) = \frac{U}{T=2\pi \times 5/c} = 1.7 \times 10^8$$

$$u_c (\text{keV}) = \frac{3}{2} \frac{6.85 \times 10^{-19} \times 2.998 \times 10^8 \times 1957^3}{5} = \mathbf{0.44}$$

$$\langle Nu^2 \rangle (\text{keV}^2/\text{s}) = \frac{55}{24\sqrt{3}} 1.7 \times 10^8 \times 0.44 = 0.99 \times 10^8$$

$$\langle N \rangle (\text{photons/s}) \sim 3.2 \times 1.7 \times 10^8 / 0.44 = \mathbf{1.2 \times 10^9}$$

### 3. Stochasticity of photoemission(7)

SPring-8 parameter is the next example.

<Case-2 E=8 GeV, ρ=40m>

$$U(\text{keV}) = \frac{88.5 \times 8^4}{40} = 9.1 \times 10^3, \quad P(\text{keV/s}) = \frac{U}{T=2\pi \times 40/c} = 1.1 \times 10^{10}$$

$$u_c (\text{keV}) = \frac{3}{2} \frac{6.85 \times 10^{-19} \times 2.998 \times 10^8 \times 15656^3}{40} = \mathbf{29.6}$$

$$\langle Nu^2 \rangle (\text{keV}^2/\text{s}) = \frac{55}{24\sqrt{3}} 1.1 \times 10^{10} \times 29.6 = 4.31 \times 10^{11}$$

$$\langle N \rangle (\text{photons/s}) \sim 3.2 \times 1.1 \times 10^{10} / 29.6 = \mathbf{1.2 \times 10^9}$$

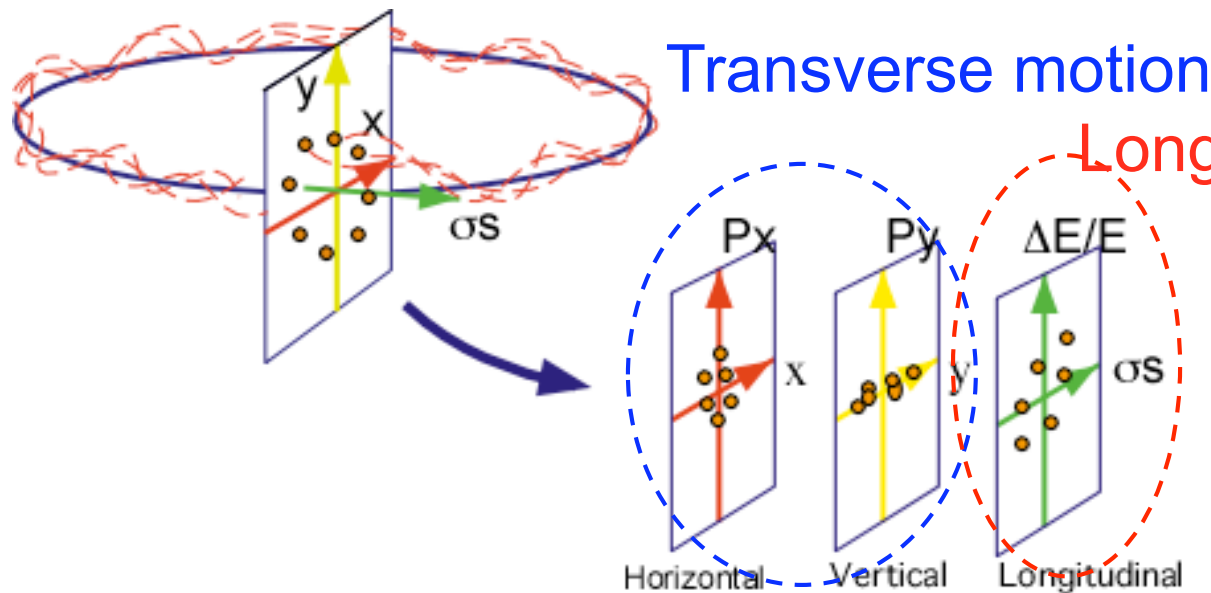
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# 4. Distribution of circulating electron beam(1)

6D-phase space volume of a single electron,  $\epsilon_{s6}$  comprises of canonical variables  $(x, px(x'), y, py(y'), t, ps(\Delta E/E))$ .  
 In an ideal case, 6D-phase space volume can be written by the product of areas of the three orthogonal 2D spaces  $\epsilon_{sz}$ .

$z=x,y,s$



$$\epsilon_{s6} = \epsilon_{sx} \epsilon_{sy} \epsilon_{ss}$$

These are invariants of the motion for a **conservative system**.

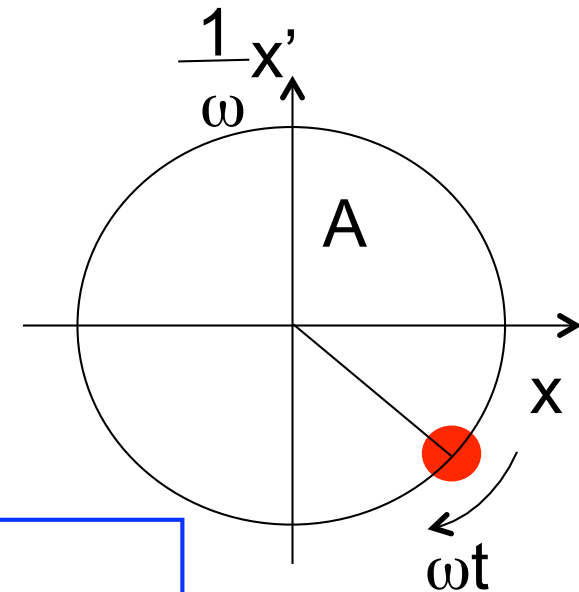
# 4. Distribution of circulating electron beam(2)

You can understand the **relation between the 2D phase space and the emittance** by using a simple harmonic oscillator.

$$x = A \cdot \cos(\omega t + \phi_0)$$
$$\frac{dx}{dt} = x' = -A \cdot \omega \sin(\omega t + \phi_0)$$
$$\longrightarrow \frac{1}{\omega} \frac{dx}{dt} = -A \cdot \sin(\omega t + \phi_0)$$

$$\text{Action} = A, \text{ Angle} = \phi = \omega t + \phi_0,$$

$$\text{Invariant} = x^2 + (x'/\omega)^2 = A^2 = \frac{\text{circle area}}{\pi} = \epsilon_{sx}$$

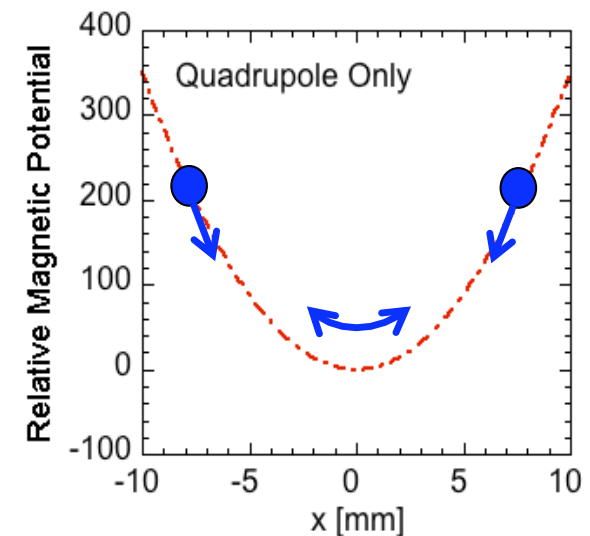


# 4. Distribution of circulating electron beam(3)

We need **potential wells to stabilize three orthogonal oscillation modes** to keep electron beam in a storage ring.

**Quadrupole magnets** generate the adequate potential wells for two transversal oscillation modes, which are called **betatron oscillations** in the horizontal and vertical planes.

**RF acceleration electric field** generates the adequate potential well for longitudinal oscillation mode, which is called a synchrotron oscillation.



# 4. Distribution of circulating electron beam(4)

SR light properties reflects the 3×2D phase space distribution of circulating electrons.

We use the following **three ensemble-averaged emittances** to express beam distribution in three orthogonal phase spaces.

$$\begin{aligned}\langle \varepsilon_{s6} \rangle &= \langle \varepsilon_{sx} \rangle \langle \varepsilon_{sy} \rangle \langle \varepsilon_{ss} \rangle \\ &= \varepsilon_x \varepsilon_y \varepsilon_s,\end{aligned}$$

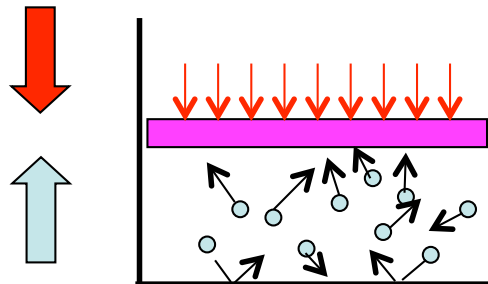
$\varepsilon_x$  : horizontal emittance (m rad)

$\varepsilon_y$  : vertical emittance (m rad)

$\varepsilon_s$  : longitudinal emittance (m rad)

# 4. Distribution of circulating electron beam(5)

Width of the Gaussian distribution of  $N$  circulating electrons is determined by the **dynamical equilibrium between the radiation excitation and damping.**



# 4. Distribution of circulating electron beam(6)

Remember the invariant of a harmonic oscillator.

$$\frac{d\langle A^2 \rangle}{dt} = \frac{d\varepsilon_z}{dt} \quad z = x, y, s.$$

**Equilibrium condition:**

$$\frac{d\langle A^2 \rangle}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\langle (A + \Delta A)^2 - A^2 \rangle}{\Delta t} = \lim_{\Delta t \rightarrow 0} 2 \frac{\langle A \Delta A \rangle}{\Delta t} + \lim_{\Delta t \rightarrow 0} \frac{\langle \Delta A^2 \rangle}{\Delta t} = 0$$

**Damping term**  
Averaged energy dissipation

**Excitation term**  
Quantum effect

# 4. Distribution of circulating electron beam(7)

Energy spread  $\sigma_{\Delta E}$

$$\lim_{\Delta t \rightarrow 0} 2 \frac{\langle A \Delta A \rangle}{\Delta t} = -2 \frac{\langle A^2 \rangle}{\tau_\varepsilon}, \quad \tau_\varepsilon = \frac{E \times T}{U},$$

$$\lim_{\Delta t \rightarrow 0} \frac{\langle \Delta A^2 \rangle}{\Delta t} = \langle N u^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c.$$

Since  $\sigma_{\Delta E}$  is **not the emittance**, phase average factor should be considered,  $\sigma_{\Delta E}^2 = \frac{\langle A^2 \rangle}{2}$ ,

$$\sigma_{\Delta E} = \sqrt{\frac{55}{96\sqrt{3}} E \times u_c}, \quad \sigma_{\Delta E/E} = \sqrt{\frac{55}{96\sqrt{3}} \frac{u_c}{E}}.$$

# 4. Distribution of circulating electron beam(8)

Bunch length  $\sigma_\tau$

$\sigma_\tau$  has two components;

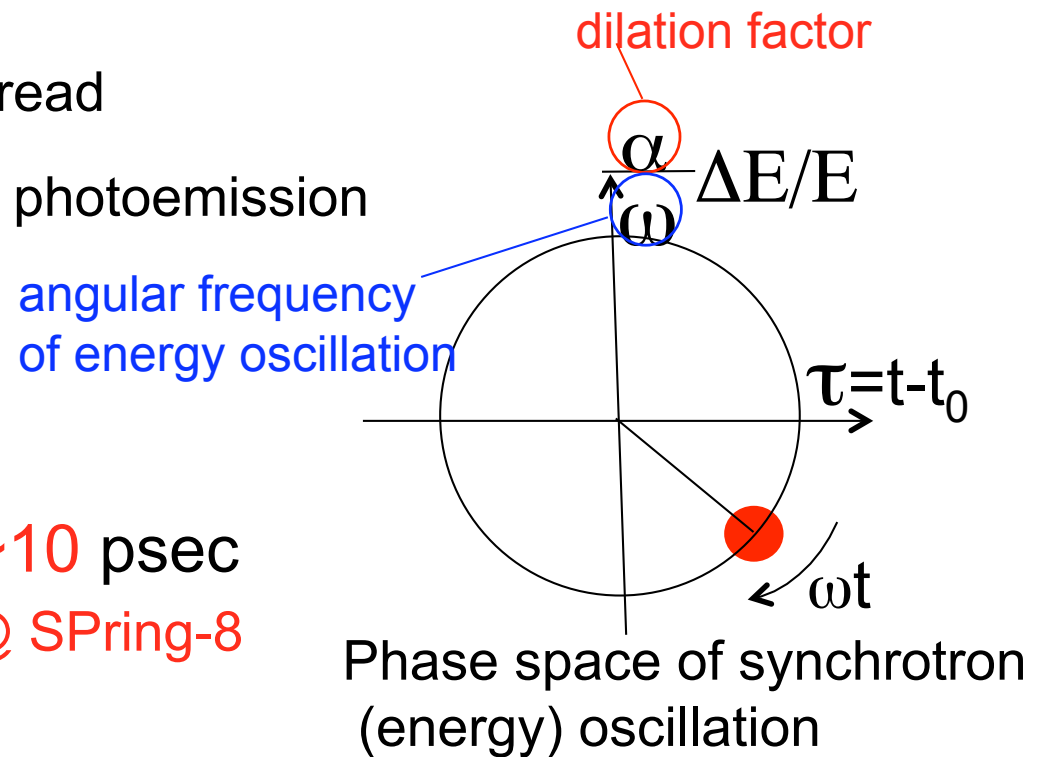
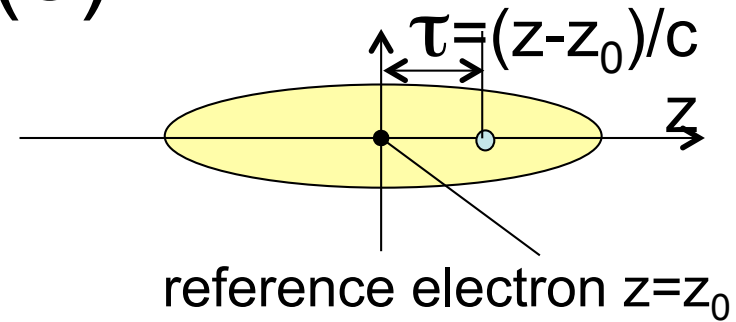
$\sigma_{\tau 1}$  = time spread by energy spread

$\sigma_{\tau 2}$  = time spread by stochastic photoemission

$$\sigma_{\tau 1} \gg \sigma_{\tau 2}$$

$$\sigma_\tau = \frac{\alpha}{\omega} \sigma_{\Delta E/E} = \sim 10 \text{ psec} @ \text{SPring-8}$$

$10^{-4}$ 
 $10^{-3}$ 
 $10^4$



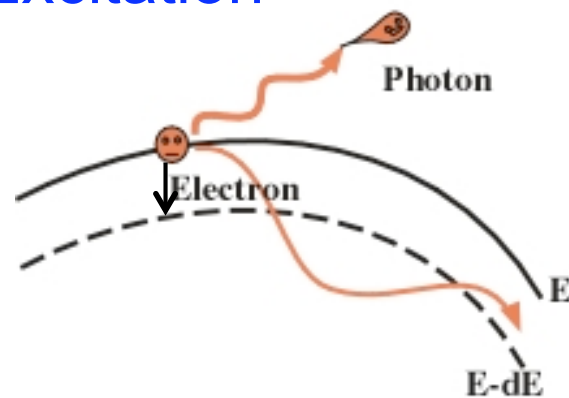
# 4. Distribution of circulating electron beam(9)

Horizontal emittance  $\epsilon_x$

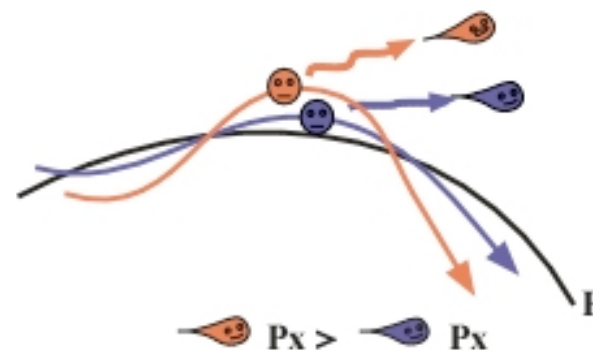
**Excitation:** due to the discrete energy jump + energy dispersion

**Damping:** due to the decrease of transverse momentum by the photoemission + acceleration along the running direction

Excitation



Damping



# 4. Distribution of circulating electron beam(10)

For the typical magnetic lattice structure (Chasman Green: CG) based storage ring, the horizontal minimum emittance is written by

$$\begin{aligned}\epsilon_{x \text{ min}} @\text{general} &\sim \frac{1}{2} \epsilon_{x \text{ min}} @\text{achromat} \\ &= \frac{C_q \gamma^2}{8\sqrt{15} J_x} \theta_b^3\end{aligned}$$

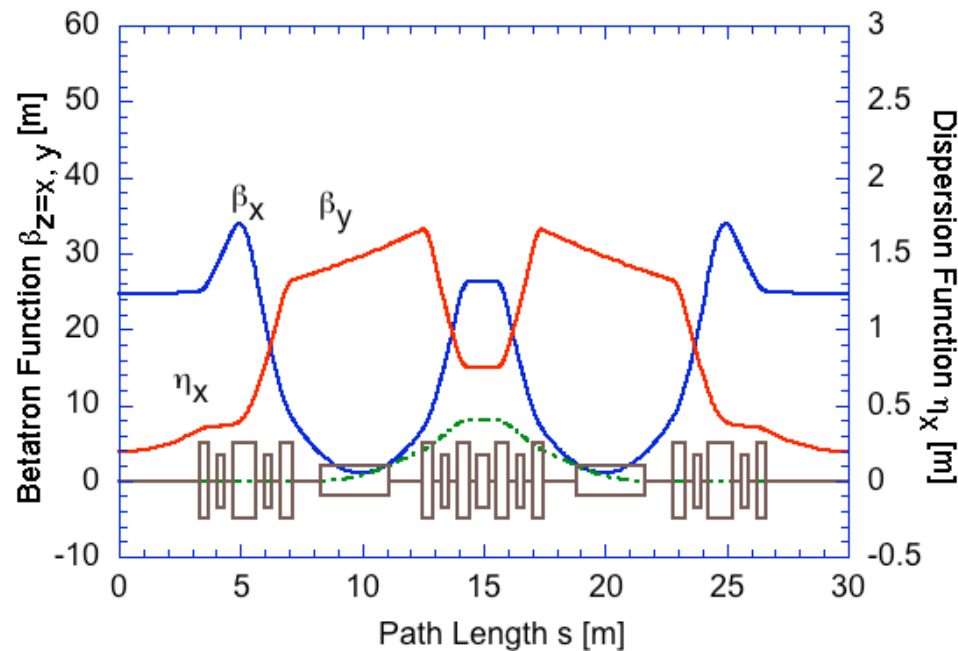
$C_q$ : Quantum constant  $3.832 \times 10^{-13}$  (m)

$\theta_b$ : Deflection angle of a single bending magnet (rad)

$J_x$ : Horizontal damping partition number  $\sim 1$

# 4. Distribution of circulating electron beam(11)

Chasman-Green (CG) lattice is the most popular magnet cell structure for a low emittance SR source, where a achromatic arc is composed of a pair of bending magnets.



2 bends: CG or CBA

3 bends: TBA

4 bends: QBA

$$\epsilon_x \text{ min} \propto \theta_b^3$$

$\theta_b$  smaller  
with same  
cell No.

# 4. Distribution of circulating electron beam(12)

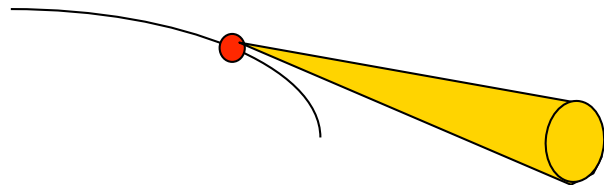
Vertical emittance  $\epsilon_y$

$\epsilon_y$  has two components;

$\epsilon_{y1}$  = vertical emittance by stochastic photoemission

$\epsilon_{y2}$  = vertical emittance by HV coupling

Usually,  $\epsilon_{y1} \ll \epsilon_{y2}$



The Angular divergence  
In the vertical plane is  $\sim 1/\gamma$

1-GeV storage ring

$\gamma = 1000/0.511 \sim 2000$

$1/\gamma \sim 5 \times 10^{-4}$

Principally,  $\epsilon_y = 1/1000 \epsilon_x$   
is possible

# 4. Distribution of circulating electron beam(13)

Vertical emittance is determined by magnetic error components mixing the horizontal and vertical betatron oscillation. The main sources are vertical misalignments of sextupole magnets and rotational errors of quadrupole magnets.

The effect of these error fields can be corrected to **0.1 % level** by the combination of beam response analysis and skew quadrupole corrector magnets.

***One-dimensional diffraction limited X-ray beam is now available***

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## 5. Approach to coherent X-rays

Here, how to achieve coherent X-rays will be discussed. The content of this part will be ready until the lecture.